THE DRESSED PHOTON IN THE PHOTOELECTRIC EFFECT

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Abstract

We solve the photoelectric problem with the dressed photon. The dressed photon is composed from the electron-positron pair. The solution uses the photon propagator with the radiative correction to the photon.

1 Introduction

The photoelectric effect is a quantum electromagnetic phenomenon in which electrons are emitted from matter after the absorption of energy from electromagnetic radiation. Frequency of radiation must be above a threshold frequency, which is specific to the type of surface and material. No electrons are emitted with a frequency below of the threshold. The photoelectric effect was theoretically explained by Einstein in his paper in 1905 (Einstein, 1905; 1965) and the term "light quanta" called "photons" was introduced by chemist G. N. Lewis, in 1926. Einstein writes (Einstein, 1905; 1965): *In accordance with the assumption to be considered here, the energy of light ray spreading out from point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.*

The linear dependence on the frequency was experimentally determined in 1915, when Robert Andrews Millikan showed that Einstein formula

$$\hbar\omega = \frac{mv^2}{2} + W \tag{1}$$

was correct. Here, $\hbar\omega$ is the energy of the impinging photon, v is the electron velocity measured by the magnetic spectrometer and W is the work function of concrete material.

The work function for Aluminium is 4.3 eV, for Beryllium 5.0 eV, for Lead 4.3 eV, for Iron 4.5 eV, and so on (Rohlf, 1994). The work function concerns the surface photoelectric effect, where the photon is absorbed by an electron in a band. The theoretical determination of the work function is the problem of the solid state physics. On the other hand, there is the so called atomic photoeffect (Amusia, 1987; Berestetzky et al., 1989), where the ionization energy plays the role of the work function. The system of the ionization energies is involved in the tables of the solid state physics.

The formula (1) is the law of conservation of energy. The classical analogue of the equation (1) is the motion of the Robins ballistic pendulum in the resistive medium.

The idea of the existence of the Compton effect is also involved in the Einstein article. He writes (Einstein, 1905; 1965): *The possibility should not be excluded, however, that electrons might receive their energy only in part from the light quantum.* However, Einstein was not sure, a priori, that his idea of such process is realistic. Only Compton proved the reality of the Einstein statement.

At energies $\hbar\omega < W$, the photoeffect is not realized. However, the photo-conductivity is the real process. The photoeffect is realized only in medium and with low energy photons, but with energies $\hbar\omega > W$, which gives the Compton effect negligible. . For $\hbar\omega \gg W$ the photoeffect is negligible in comparison with the Compton effect. At the same time it is necessary to say that the Feynman diagram of the Compton effect cannot be reduced to the Feynman diagram for photoeffect. In case of the high energy gamma rays, it is possible to consider the process called photoproduction of elementary particles on protons in LHC, or, photo-nuclear reactions in nuclear physics (Levinger, 1960). Such processes are energetically far from the photoelectric effect in solid state physics.

Eq. (1) represents so called one-photon photoelectric effect, which is valid for very weak electromagnetic waves. At present time of the laser physics, where the strong electromagnetic intensity is possible, we know that so called multiphoton photoelectric effect is possible. Then, instead of equation (1) we can write

$$\hbar\omega_1 + \hbar\omega_2 + ...\hbar\omega_n = \frac{mv^2}{2} + W. \qquad (2)$$

The time lag between the incidence of radiation and the emission of a photoelectron is very small, less than 10^{-9} seconds.

The ejected electron has the final plane wave

$$\psi_{\mathbf{q}} = \frac{1}{\sqrt{V}} e^{i\mathbf{q}\cdot\mathbf{x}}, \quad \mathbf{q} = \frac{\mathbf{p}}{\hbar}, \qquad (3)$$

where $\mathbf{p}$ is the momentum of the ejected electron.

The probability of the emission of electron by the electromagnetic wave is of the well-known form (Davydov, 1976):

$$dP = \frac{e^2 p}{8\pi^2 \varepsilon_0 \hbar m \omega} \left| \int e^{i(\mathbf{k}-\mathbf{q})\cdot\mathbf{x}} (\mathbf{e}\cdot\nabla)\psi_0 dxdydz \right|^2 d\Omega = C|J|^2 d\Omega, \qquad (4)$$

where the interaction for absorption of the electromagnetic wave is normalized to *one photon in the unit volume*, $\mathbf{e}$ is the polarization of the impinging photon, ε_0 is the dielectric constant of vacuum, ψ_0 is the basic state of and atom. We have denoted the integral in $||$ by J and the constant before $||$ by C.

Let us consider the case with electrons in magnetic field as an analog of the Landau diamagnetism. So, we take the basic function ψ_0 for one electron in the lowest Landau level, as

$$\psi_0 = \left(\frac{m\omega_c}{2\pi\hbar}\right)^{1/2} \exp\left(-\frac{m\omega_c}{4\hbar}(x^2+y^2)\right), \tag{5}$$

which is solution of the Schrödinger equation in the magnetic field with potentials $\mathbf{A} = (-Hy/2, -Hx/2, 0,)$, (Drukarev, 1988):

$$\left[\frac{p_x^2}{2m} + \frac{p_y^2}{2m} - \frac{m}{2}\left(\frac{\omega_c}{2}\right)^2 (x^2+y^2)\right]\psi = E\psi. \tag{6}$$

We have supposed that the motion in the z-direction is zero and it means that the wave function $\exp[(i/\hbar)p_z z] = 1$.

So, the main problem is to calculate the integral

$$J = \int e^{i(\mathbf{K}\cdot\mathbf{x})}(\mathbf{e}\cdot\nabla)\psi_0 dxdydz; \quad \mathbf{K} = \mathbf{k} - \mathbf{q}. \tag{7}$$

with the basic Landau function ψ_0 given by the equation (5).

Operator $(\hbar/i)\nabla$ is Hermitean and it means we can rewrite the last integrals as follows:

$$J = \frac{i}{\hbar}\mathbf{e}\cdot\int\left[\left(\frac{\hbar}{i}\nabla\right)e^{i(\mathbf{K}\cdot\mathbf{x})}\right]^* \psi_0 dxdydz, \tag{8}$$

which gives

$$J = i\mathbf{e}\cdot\mathbf{K}\int e^{-i(\mathbf{K}\cdot\mathbf{x})}\psi_0 dxdydz, \tag{9}$$

The integral in eq. (9) can be transformed using the cylindrical coordinates with $dxdydz = \varrho d\varrho d\varphi dz$, $\varrho^2 = x^2 + y^2$, which gives for vector $\mathbf{K}$ fixed on the axis z with $\mathbf{K}\cdot\mathbf{x} = Kz$ and with physical condition $\mathbf{e}\cdot\mathbf{k} = 0$, expressing the physical situation where polarization is perpendicular to the direction of the wave propagation. So,

$$J = (i)(\mathbf{e}\cdot\mathbf{q})\int_0^\infty \varrho d\varrho \int_{-\infty}^\infty dz \int_0^{2\pi} d\varphi e^{-iKz}\psi_0. \tag{10}$$

Using

$$\psi_0 = A\exp\left(-B\varrho^2\right); \quad A = \left(\frac{m\omega_c}{2\pi\hbar}\right)^{1/2}; \quad B = \frac{m\omega_c}{4\hbar}, \tag{11}$$

the integral (10) is then

$$J = (-\pi i)\frac{A}{B}(\mathbf{e}\cdot\mathbf{q})\int_{-\infty}^\infty e^{-iKz}dz = (-\pi i)\frac{A}{B}(\mathbf{e}\cdot\mathbf{q})(2\pi)\delta(K). \tag{12}$$

Then,

$$dP = C|J|^2 d\Omega = 4\pi^4\frac{A^2}{B^2}C(\mathbf{e}\cdot\mathbf{q})^2\delta^2(K)d\Omega. \tag{13}$$

Now, let be the angle Θ between direction $\mathbf{k}$ and direction $\mathbf{q}$, and let be the angle Φ between planes $(\mathbf{k},\mathbf{q})$ and $(\mathbf{e},\mathbf{k})$. Then,

$$(\mathbf{e}\cdot\mathbf{q})^2 = q^2\sin^2\Theta\cos^2\Phi. \tag{14}$$

So, the differential probability of the emission of photons from the graphene (Pardy, 2010) in the strong magnetic field is as follows:

$$dP = \frac{4e^2 p}{\pi\varepsilon_0 m^2\omega\omega_c}\left[q^2\cos^2\Theta\sin^2\Phi\right]\delta^2(K)d\Omega; \quad \omega_c = \frac{|e|H}{mc}. \tag{15}$$

We can see that our result differs form the result for the original photoelectric effect which involves still the term

$$\frac{1}{(1-\frac{v}{c}\cos\Theta)^4}, \tag{16}$$

which means that the most intensity of the classical photoeffect is in the direction of the electric vector of the electromagnetic wave ($\Phi = \pi/2, \Theta = 0$). While the nonrelativistic solution of the photoeffect in case of the Coulomb potential was performed by Stobbe (1930) and the relativistic calculation by Sauter (Sauter, 1931), the general magnetic photoeffect (with electrons moving in the magnetic field and forming atom) was not still performed in a such simple form. The delta term $\delta\cdot\delta$ represents the conservation law $|\mathbf{k}-\mathbf{q}| = 0$ in our approximation.

So, we have calculated only the process which can be approximated by the Schrödinger equation for an electron orbiting in magnetic field.

The photoeffect with the dressed photon is the process, where the dressed photon is taken with the radiative correction in the form of the virtual electron-positron pair.

We have shown that such approach to the photon leads to the modification he photon propagator. According to Dittrich (1978) and Schwinger (1973), the photon propagator with radiative correction is in the momentum representation of the form:

$$\tilde{D}(k) = D(k) + \delta D(k), \tag{17}$$

or,

$$\tilde{D}(k) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\epsilon} +$$

$$+\int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{|\mathbf{k}|^2 - n^2(k^0)^2 + \frac{M^2c^2}{\hbar^2} - i\epsilon}, \quad 18)$$

where the last term in equation (18) is derived on the virtual photon condition

$$|\mathbf{k}|^2 - n^2(k^0)^2 = -\frac{M^2c^2}{\hbar^2}, \tag{19}$$

where n is the index of refraction of the medium. The weight function $a(M^2)$ has been derived in the following form (Dittrich, 1978; Schwinger, 1973):

$$a(M^2) = \frac{\alpha}{3\pi}\frac{1}{M^2}\left(1+\frac{2m^2}{M^2}\right)\left(1-\frac{4m^2}{M^2}\right)^{1/2}. \tag{20}$$

The x-representation of $D(k)$ in eq. (18) is as follows:

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x-x')} D(k). \quad (21)$$

Or,

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4} \frac{e^{ik(x-x')}}{|\mathbf{k}^2| - n^2(k^0)^2 - i\epsilon} =$$

$$= \frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|} e^{-i\omega|t-t'|}. \quad (22)$$

Now, with regard to the definition of x-representation (21) and (22) of the $D_+(x-x')$, we get the x-representation of the δD_+ in the following form:

$$\delta D_+(x-x') = \frac{i}{c}\frac{1}{4\pi^2}\int_{4m^2}^\infty dM^2 a(M^2)\times$$

$$\times \int d\omega \, \frac{\sin[\frac{n^2\omega^2}{c^2} - \frac{M^2c^2}{\hbar^2}]^{1/2}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|} e^{-i\omega|t-t'|}. \quad (23)$$

The function (23) differs from the the original function D_+ especially by the factor

$$\gamma = \left(\frac{\omega^2 n^2}{c^2} - \frac{M^2c^2}{\hbar^2}\right)^{1/2} \quad (24)$$

and by the additional mass-integral which involves the radiative corrections to the original photon processes. It was easily shown in case of the Čerenkov effect by author (Pardy, 1994).

So, to involve the photoelectric effect with the dressed photon with electron positron pair we replace the wave function of photon $\exp(i\mathbf{k}\cdot\mathbf{x})$ by the function involving the radiative correction factor as follows:

$$e^{i\mathbf{k}\cdot\mathbf{x}} \rightarrow \int_{4m^2}^\infty dM^2 a(M^2) e^{i\boldsymbol{\kappa}\cdot x}, \quad (25)$$

where $\boldsymbol{\kappa}\cdot\mathbf{x} = \gamma|k||x|\cos\varphi$.

The probability of the emission of electron by the electromagnetic wave is given by eq. (4).

So, the main problem is to calculate the integral

$$J = \int e^{i(\boldsymbol{\kappa}\cdot\mathbf{x})}(\mathbf{e}\cdot\nabla)\psi_0 dxdydz; \quad \mathbf{K} = \boldsymbol{\kappa} - \mathbf{q}. \quad (26)$$

with the basic Landau function ψ_0 given by the equation (5).

Then, the differential probability of the emission of photons from the plane in the strong magnetic field is as follows:

$$dP = \frac{4e^2 p}{\pi\varepsilon_0 m^2 \omega\omega_c}\int_{4m^2}^\infty dM^2 a(M^2)\left[q^2\cos^2\Theta\sin^2\Phi\right]\delta^2(K)d\Omega; \quad \omega_c = \frac{|e|H}{mc}. \quad (27)$$

We can see that our result differs form the result (15) by the mass term and by the argument in the δ-function. The delta term $\delta\cdot\delta$ represents the conservation law $|\boldsymbol{\kappa}-\mathbf{q}| = 0$

in our approximation. The dressed photon was here considered as the photon composed from the electron-positron pair. It is not excluded that the photoelectric experiments with the dressed photon is related to the experiments with the Vavilov-Cherenkov phenomenon in metal nanofilms (Pardy, 2007, 2010, 2011; Zuev, 2009).

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