Universal Reference Frame

Alejandro A. Torassa

(2013) Buenos Aires, Argentina
atorassa@gmail.com

Abstract

In classical mechanics, this paper presents the universal reference frame.

Universal Reference Frame

The universal reference frame is a reference frame fixed to the center of mass of the universe.

The position $\mathring{\mathbf{r}}_a$, the velocity $\mathring{\mathbf{v}}_a$, and the acceleration $\mathring{\mathbf{a}}_a$ of a particle A of mass m_a relative to the universal reference frame $\mathring{\mathrm{S}}$, are given by:

$$\mathring{\mathbf{r}}_a = \int\int (\mathbf{F}_a/m_a)\; dt\; dt$$

$$\mathring{\mathbf{v}}_a = \int (\mathbf{F}_a/m_a)\; dt$$

$$\mathring{\mathbf{a}}_a = (\mathbf{F}_a/m_a)$$

where $\mathbf{F}_a$ is the net force acting on particle A.

From the above equations the following equations are obtained:

$m_a\mathring{\mathbf{r}}_a - \int\int \mathbf{F}_a\; dt\; dt = 0$	$\rightarrow$	$\tfrac{1}{2}\, m_a\mathring{\mathbf{r}}_a^2 - \tfrac{1}{2}\,(\int\int \mathbf{F}_a\; dt\; dt)^2 = 0$
$\downarrow$		$\downarrow$
$m_a\mathring{\mathbf{v}}_a - \int \mathbf{F}_a\; dt = 0$	$\rightarrow$	$\tfrac{1}{2}\, m_a\mathring{\mathbf{v}}_a^2 - \int \mathbf{F}_a\; d\mathring{\mathbf{r}}_a = 0$
$\downarrow$	$\nearrow$	$\downarrow$
$m_a\mathring{\mathbf{a}}_a - \mathbf{F}_a = 0$	$\rightarrow$	$\tfrac{1}{2}\, m_a\mathring{\mathbf{a}}_a^2 - \tfrac{1}{2}\,(\mathbf{F}_a^2/m_a) = 0$

Reference Frame

The position $\mathring{\mathbf{r}}_a$, the velocity $\mathring{\mathbf{v}}_a$, and the acceleration $\mathring{\mathbf{a}}_a$ of a particle A of mass m_a relative to a reference frame S, are given by:

$$\mathring{\mathbf{r}}_a = \mathbf{r}_a + \mathring{\mathbf{r}}_S$$

$$\mathring{\mathbf{v}}_a = \mathbf{v}_a + \mathring{\omega}_S \times \mathbf{r}_a + \mathring{\mathbf{v}}_S$$

$$\mathring{\mathbf{a}}_a = \mathbf{a}_a + 2\,\mathring{\omega}_S \times \mathbf{v}_a + \mathring{\omega}_S \times (\mathring{\omega}_S \times \mathbf{r}_a) + \mathring{\alpha}_S \times \mathbf{r}_a + \mathring{\mathbf{a}}_S$$

where $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{a}_a$ are the position, the velocity, and the acceleration of particle A relative to the reference frame S; $\mathring{\mathbf{r}}_S$, $\mathring{\mathbf{v}}_S$, $\mathring{\mathbf{a}}_S$, $\mathring{\omega}_S$, and $\mathring{\alpha}_S$ are the position, the velocity, the acceleration, the angular velocity, and the angular acceleration of the reference frame S relative to the universal reference frame $\mathring{\text{S}}$.

The position $\mathring{\mathbf{r}}_S$, the velocity $\mathring{\mathbf{v}}_S$, the acceleration $\mathring{\mathbf{a}}_S$, the angular velocity $\mathring{\omega}_S$, and the angular acceleration $\mathring{\alpha}_S$ of a reference frame S fixed to a particle S relative to the universal reference frame $\mathring{\text{S}}$, are given by:

$$\mathring{\mathbf{r}}_S = \int\!\int (\mathbf{F}_0/m_s)\; dt\; dt$$

$$\mathring{\mathbf{v}}_S = \int (\mathbf{F}_0/m_s)\; dt$$

$$\mathring{\mathbf{a}}_S = (\mathbf{F}_0/m_s)$$

$$\mathring{\omega}_S = \left|(\mathbf{F}_1/m_s - \mathbf{F}_0/m_s)/(\mathbf{r}_1 - \mathbf{r}_0)\right|^{1/2}$$

$$\mathring{\alpha}_S = d(\mathring{\omega}_S)/dt$$

where $\mathbf{F}_0$ is the net force acting on the reference frame S in a point 0, $\mathbf{F}_1$ is the net force acting on the reference frame S in a point 1, $\mathbf{r}_0$ is the position of the point 0 relative to the reference frame S (the point 0 is the center of mass of particle S and the origin of the reference frame S) $\mathbf{r}_1$ is the position of the point 1 relative to the reference frame S (the point 1 does not belong to the axis of rotation) and m_s is the mass of particle S (the vector $\mathring{\omega}_S$ is along the axis of rotation)

On the other hand, the position $\mathring{\mathbf{r}}_S$, the velocity $\mathring{\mathbf{v}}_S$, and the acceleration $\mathring{\mathbf{a}}_S$ of a reference frame S relative to the universal reference frame $\mathring{\text{S}}$ are related to the position $\mathbf{r}_{cm}$, the velocity $\mathbf{v}_{cm}$, and the acceleration $\mathbf{a}_{cm}$ of the center of mass of the universe relative to the reference frame S.

Kinetic Force

The kinetic force $\mathbf{K}_{ab}$ exerted on a particle A of mass m_a by another particle B of mass m_b, caused by the interaction between particle A and particle B, is given by:

$$\mathbf{K}_{ab} = \frac{m_a m_b}{m_{cm}} (\mathring{\mathbf{a}}_a - \mathring{\mathbf{a}}_b)$$

where m_{cm} is the mass of the center of mass of the universe, $\mathring{\mathbf{a}}_a$ and $\mathring{\mathbf{a}}_b$ are the accelerations of particles A and B relative to the universal reference frame $\mathring{\mathrm{S}}$.

From the above equation it follows that the net kinetic force $\mathbf{K}_a$ acting on a particle A of mass m_a, is given by:

$$\mathbf{K}_a = m_a \mathring{\mathbf{a}}_a$$

where $\mathring{\mathbf{a}}_a$ is the acceleration of particle A relative to the universal reference frame $\mathring{\mathrm{S}}$.

From page [1], we have:

$$m_a \mathring{\mathbf{a}}_a - \mathbf{F}_a = 0$$

That is:

$$\mathbf{K}_a - \mathbf{F}_a = 0$$

Therefore, the total force $(\mathbf{K}_a - \mathbf{F}_a)$ acting on a particle A is always in equilibrium.

Bibliography

A. Einstein, Relativity: The Special and General Theory.

E. Mach, The Science of Mechanics.

R. Resnick and D. Halliday, Physics.

J. Kane and M. Sternheim, Physics.

H. Goldstein, Classical Mechanics.

L. Landau and E. Lifshitz, Mechanics.