Deformed Quantum Field Theory, Thermodynamics at Low and High Energies, and Gravity. I

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PACS: 03.65, 05.20
Keywords: quantum field theory with UV cutoff, gravitational thermodynamics,deformed gravity

Abstract

The present work is a natural continuation of the previous paper arXiv: 0911.5597. In this work, within the scope of the Generalized Uncertainty Principle, a model of the high energy deformation for a particular case of Einstein's equations is developed. In the process a thermodynamic description of General Relativity is used. And the deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition. The possibility for the high energy deformation of Einstein's equations within the scope of both equilibrium thermodynamics and non-equilibrium thermodynamics is examined.

1 Introduction

The present work is a natural continuation of the previous paper arXiv: 0911.5597. It should be noted that there is a certain discord between the modern development of quantum mechanics and quantum field theory, on the one hand, and gravity, on the other hand. In the last decade the researchers have come to an understanding that in the process of studies into the physics of the Early Universe (extremely high – Plancks energies) the fundamental physical theories, in particular quantum mechanics and quantum field theory, should be changed. It is inevitable that in these theories a fundamental length should be introduced. In so doing the correspondence principle should

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be followed without fail: at well-known low energies the theories involving the fundamental length must present the conventional quantum mechanics and quantum field theory with a high precision. The idea that a quantum theory at the Planck scales must involve the fundamental length has been put forward in the works devoted to a string theory fairly a long time ago [1]. But since it is still considered to be a tentative theory, some other indications have been required. Fortunately, by the present time numerous publications have suggested the appearance of the fundamental length in the Early Universe with the use of various approaches [2]–[5]. Of particular importance is the work [2], where on the basis of a simple gedanken experiment it is demonstrated that, with regard to the gravitational interactions (Plancks scales)exhibited in the Early Universe only, the Heisenberg Uncertainty Principle should be extended to the Generalized Uncertainty Principle [1]–[5]that in turn is bound to bring forth the fundamental length on the order of Plancks length. The advent of novel theories in physics of the Early Universe is associated with the introduction of new parameters, i.e. with a deformation of the well-known theories. The deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition [6]. Of course, in this case Heisenberg Algebra is subjected to the corresponding deformation too. Such a deformation may be based on the Generalized Uncertainty Principle (GUP) [7]–[9] as well as on the density matrix deformation [10]–[18].

At the same time, the above-mentioned new deformation parameters so far have not appeared in gravity despite the idea that they should. The situation is that no evident efforts have been undertaken to develop the high-energy (Plancks scale) gravity deformations including the deformation parameters introduced in a Quantum Theory of the Early Universe.

In this paper, with GUP held true, the possibility for the high-energy gravity deformation is considered for a specific case of Einstein's equations. As this takes place, the parameter α appearing in the Quantum Field Theory (QFT) with the UV cutoff (fundamental length) produced by the density matrix deformation is used. There is no discrepancy of any kind as the deformation parameter in the GUP-produced Heisenberg algebra deformation is quite naturally expressed in terms of α, and this will be shown later (Section 2). Besides, by its nature, α is better applicable to study the high-energy deformation of General Relativity because it is small, dimensionless (making series expansion more natural), and the corresponding representation f Einstein's equations in its terms or its deformation appear simple. Structurally, the paper is as follows. In Sections 2 and 3 the approaches to the deformation of a quantum theory at the Planck scales are briefly reviewed. In Section 4 together with various inferences a strategy is suggested to study possible high energy generalizations (deformations) of General Relativity. Actually, Section 4 represents a short variant of Section 5 given in arXiv: 0911.5597 . New results are presented in Sections 5 and 6. A thermodynamic description of General Relativity is used. The possibility for the high energy deformation of Einstein's equations is discussed within the scope of both equilibrium thermodynamics and non-equilibrium thermodynamics. In the latter case the approach is contemplated only in terms of a nature of the cosmological constant.

2 Quantum Theory at Planck's Scale

In the last few years the researchers have come to the understanding that studies of the Early Universe physics (extremely high Plancks energies) necessitate changes in the fundamental physical theories, specifically quantum mechanics and quantum field theory. Inevitably a fundamental length should be involved in these theories [4]–[8]. This idea has been first suggested by a string theory [1]. But it is still considered to be a tentative theory without the experimental status and merely an attractive model. However, the fundamental length has been involved subsequently in more simple and natural considerations [2].

The main approach to framing of Quantum Mechanics with fundamental length (QMFL) and Quantum Field Theory with fundamental length (QFTFL) (or with Ultraviolet (UV) cutoff) is that associated with the Generalized Uncertainty Principle (GUP) [1]–[9]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_p^2 \frac{\triangle p}{\hbar}. \qquad (1)$$

with the corresponding Heisenberg algebra deformation produced by this principle [7]–[9].

Besides, in the works by the author [10]–[19] an approach to the construction of QMFL has been developed with the help of the deformed density matrix, the density matrix deformation in QMFL being a starting object called the density pro-matrix $\rho(\alpha)$ and deformation parameter (additional parameter)

$\alpha = l_{min}^2/x^2$,$0 < \alpha \leq 1/4$ where x is the measuring scale and $l_{min} \sim l_p$ [10],[11].

The explicit form of the above-mentioned deformation gives an exponential ansatz:

$$\rho^*(\alpha) = exp(-\alpha)\sum_i \omega_i|i><i|, \tag{2}$$

where all $\omega_i > 0$ are independent of α and their sum is equal to 1.
In the corresponding deformed Quantum Theory (denoted as QFT^α) for average values we have

$$<B>_\alpha = exp(-\alpha) <B>, \tag{3}$$

where $<B>$ - average in well-known QFT [15],[16]. All the variables associated with the considered α - deformed quantum field theory are hereinafter marked with the upper index $^\alpha$.
Note that the deformation parameter α is absolutely naturally represented as a ratio between the squared UV and IR limits

$$\alpha = (\frac{UV}{IR})^2, \tag{4}$$

where UV is fixed and IR is varying.
It should be noted [20] that in a series of the authors works [10]–[19] a minimal α-deformation of QFT has been formed. By minimal it is meant that no space-time noncommutativity was required, i.e. there was no requirement for noncommutative operators associated with different spatial coordinates

$$[X_i, X_j] \neq 0, i \neq j. \tag{5}$$

However, all the well-known deformations of QFT associated with GUP (for example, [7]–[9]) contain (5) as an element of the corresponding deformed Heisenberg algebra. Because of this, it is necessary to extend (or modify) the above-mentioned minimal α-deformation of QFT $-QFT^\alpha$ [10]–[19] to some new deformation $\widetilde{QFT}^\alpha$ compatible with GUP, as it has been noted in [20]. We can easily show that QFT parameter of deformations associated with GUP may be expressed in terms of the parameter α that has been introduced in the approach associated with the density matrix deformation. Here the notation of [21] is used. Then

$$[\vec{x}, \vec{p}] = i\hbar(1 + \beta^2\vec{p}^2 + ...) \tag{6}$$

and

$$\Delta x_{\min} \approx \hbar\sqrt{\beta} \sim l_p. \qquad (7)$$

Then from (6),(7) it follows that $\beta \sim \mathbf{1/p^2}$, and for $x_{\min} \sim l_p$, β corresponding to $x_{\min}$ is nothing else but

$$\beta \sim 1/P_{pl}^2, \qquad (8)$$

where P_{pl} is Planck's momentum: $P_{pl} = \hbar/l_p$.
In this way β is changing over the following interval:

$$\lambda/P_{pl}^2 \leq \beta < \infty, \qquad (9)$$

where λ is a numerical factor and the second member in (6) is accurately reproduced in momentum representation (up to the numerical factor) by $\alpha = l_{min}^2/l^2 \sim l_p^2/l^2 = p^2/P_{pl}^2$

$$[\vec{x}, \vec{p}] = i\hbar(1 + \beta^2\vec{p}^2 + ...) = i\hbar(1 + a_1\alpha + a_2\alpha^2 + ...). \qquad (10)$$

3 Some Inferences of Quantum Theories of the UV-cutoff

The above-mentioned deformations of a quantum field theory at Plancks scales have several important inferences. In particular, a Quantum Field Theory (corresponding Heisenberg algebra deformations) within the scope of GUP [7]–[9] suggests the high-energy quantum corrections for temperature and entropy of the black holes [26]– [33].In the recent work [21] it has been demonstrated that the Holographic Principle [22]– [25] is actually integrated in the approach. Moreover, on the assumption that the cosmological term Λ is a dynamic quantity [34]– [40], the Heisenberg Uncertainty principle derived in [37]–[40] for the pair of conjugate variables (Λ, V):

$$\Delta\Lambda\, \Delta V \sim \hbar, \qquad (11)$$

where V is the space-time volume, may be extended up to GUP [41],[42]. At least heuristically, this result may account for a giant, by a factor of $\approx 10^{122}$, discrepancy between the value of Λ calculated within the scope of conventional QFT and the experimental value [43],[44].
On the other hand, the parameter α and the corresponding deformation of a quantum field theory QFT^{α} [10]–[19] also gives a good explanation for this

discrepancy [42],[45],[46], at least within the holographic principle and for the Holographic Dark Energy Models (HDE) [47] – [51], as at the known infrared limit (IR cut-off) of the Universe $L_{IR} \approx 10^{122}$ the quantity α is just equal to $\alpha_{min} \approx 10^{122}$. Besides, an approach based on the density matrix deformation suggests a phenomenological solution for a number of problems in physics of black holes: Liouville equation modification [52] (deformation)[11][13],[18], information paradox of Hawking [53],[13],[14],[18], and calculation of quantum corrections [18]–[20] to a semiclassical Bekenstein-Hawking formula of the black hole entropy [55], [54].
It should be noted that GUP (1) may be complemented by the Generalized Uncertainty Relation in Thermodynamics (at Planck energy) [12],[18],[58]:

$$\Delta\frac{1}{T} \geq \frac{k}{\Delta U} + \eta\left(\frac{\Delta U}{E_p}\right)\frac{k_B}{E_p} + ... \tag{12}$$

where T is the ensemble temperature, U is its internal energy, k_B is the Boltzmann constant, E_p is the Planck energy.In the recently published work [59]) the black hole horizon temperature has been measured with the use of the Gedanken experiment. In the process the Generalized Uncertainty Relations in Thermodynamics (12) have been derived also. Expression (12) has been considered in the monograph [60] within the scope of the mathematical physics methods.

4 Gravitational Thermodynamics in Low and High Energy and Deformed Quantum Theory

In the last decade a number of very interesting works have been published. We can primary name the works [62]–[73], where gravitation, at least for the spaces with horizon, is directly associated with thermodynamics and the results obtained demonstrate a holographic character of gravitation. Of the greatest significance is a pioneer work [56]. For black holes the association has been first revealed in [55],[52], where related the black-hole event horizon temperature to the surface gravitation. In [72], has shown that this relation is not accidental and may be generalized for the spaces with horizon. As all the foregoing results have been obtained in a semiclassical approximation, i.e. for sufficiently low energies, the problem arises: how these results are modified when going to higher energies. In the context of this paper, the problem may be stated as follows: since we have some infra-red (IR) cutoff l_{max} and ultraviolet (UV) cutoff l_{min}, we naturally have a problem how the above-mentioned results on Gravitational Thermodynamics are changed for

$$l \rightarrow l_{min}. \tag{13}$$

According to Sections 2 and 3 of this paper, they should become dependent on the deformation parameter α. After all, in the already mentioned in Section (formula (4)) α is indicated as nothing else but

$$\alpha = \frac{l_{min}^2}{l^2}. \tag{14}$$

In fact, in several papers [26]–[32] it has been demonstrated that thermodynamics and statistical mechanics of black holes in the presence of GUP (i.e. at high energies) should be modified. To illustrate, in [31] the Hawking temperature modification has been computed in the asymptotically flat space in this case in particular. It is easily seen that in this case the deformation parameter α arises naturally. Indeed, modification of the Hawking temperature is of the following form(formula (10) in [31]):

$$T_{GUP} = (\frac{d-3}{4\pi})\frac{\hbar r_+}{2\alpha'^2 l_p^2}[1 - (1 - \frac{4\alpha'^2 l_p^2}{r_+^2})^{1/2}], \tag{15}$$

where d is the space-time dimension, and r_+ is the uncertainty in the emitted particle position by the Hawking effect, expressed as

$$\Delta x_i \approx r_+ \tag{16}$$

and being nothing else but a radius of the event horizon; α' – dimensionless constant from GUP. But as we have $2\alpha' l_p = l_{min}$, in terms of α (15) may be written in a natural way as follows:

$$T_{GUP} = (\frac{d-3}{4\pi})\frac{\hbar \alpha_{r_+}^{-1}}{\alpha' l_p}[1 - (1 - \alpha_{r_+})^{1/2}], \tag{17}$$

where α_{r_+}- parameter α associated with the IR-cutoff r_+. In such a manner T_{GUP} is only dependent on the constants including the fundamental ones and on the deformation parameter α.

The dependence of the black hole entropy on α may be derived in a similar way. For a semiclassical approximation of the Bekenstein-Hawking formula [55],[52]

$$S = \frac{1}{4}\frac{A}{l_p^2}, \tag{18}$$

where A – surface area of the event horizon, provided the horizon event has radius r_+, then $A \sim r_+^2$ and (18) is clearly of the form

$$S = \sigma \alpha_{r_+}^{-1}, \tag{19}$$

where σ is some dimensionless denumerable factor. The general formula for quantum corrections [30] given as

$$S_{GUP} = \frac{A}{4l_p^2} - \frac{\pi\alpha'^2}{4}\ln\left(\frac{A}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \text{const} , \tag{20}$$

where the expansion coefficients $c_n \propto \alpha'^{2(n+1)}$ can always be computed to any desired order of accuracy [30], may be also written as a power series in $\alpha_{r_+}^{-1}$ (or Laurent series in α_{r_+})

$$S_{GUP} = \sigma\alpha_{r_+}^{-1} - \frac{\pi\alpha'^2}{4}\ln(\sigma\alpha_{r_+}^{-1}) + \sum_{n=1}^{\infty}(c_n\sigma^{-n})\alpha_{r_+}^n + \text{const} \tag{21}$$

Note that here no consideration is given to the restrictions on the IR-cutoff

$$l \leq l_{max} \tag{22}$$

and to those corresponding the extended uncertainty principle (EUP) that leads to a minimal momentum [31]. This problem will be considered separately in further publications of the author.
A black hole is a specific example of the space with horizon. It is clear that for other horizon spaces [72] a similar relationship between their thermodynamics and the deformation parameter α should be exhibited.
Quite recently, in a series of papers, and specifically in [64]–[70], it has been shown that Einstein equations may be derived from the surface term of the GR Lagrangian, in fact containing the same information as the bulk term.
It should be noted that Einstein's equations [at least for space with horizon] may be obtained from the proportionality of the entropy and horizon area together with the fundamental thermodynamic relation connecting heat,

entropy, and temperature [56]. In fact [64]– [71], this approach has been extended and complemented by the demonstration of holographicity for the gravitational action (see also [72]).And in the case of Einstein-Hilbert gravity, it is possible to interpret Einstein's equations as the thermodynamic identity [73]:

$$TdS = dE + PdV. \tag{23}$$

The above-mentioned results in the last paragraph have been obtained at low energies, i.e. in a semiclassical approximation. Because of this, the problem arises how these results are changed in the case of high energies? Or more precisely, how the results of [56],[64]– [73] are generalized in the UV-limit? It is obvious that, as in this case all the thermodynamic characteristics become dependent on the deformation parameter α, all the corresponding results should be modified (deformed) to meet the following requirements:
(a) to be clearly dependent on the deformation parameter α at high energies;

(b) to be duplicated, with high precision, at low energies due to the suitable limiting transition;

(c) Let us clear up what is meant by the adequate high energy α-deformation of Einstein's equations (General Relativity).
The problem may be more specific.
As, according to [56],[72],[73] and some other works, gravitation is greatly determined by thermodynamics and at high energies the latter is a deformation of the classical thermodynamics, it is interesting whether gravitation at high energies (or what is the same, quantum gravity or Planck scale)is being determined by the corresponding deformed thermodynamics. The formulae (17) and (21) are elements of the high-energy α-deformation in thermodynamics, a general pattern of which still remains to be formed. Obviously, these formulae should be involved in the general pattern giving better insight into the quantum gravity, as they are applicable to black mini-holes (Planck black holes) which may be a significant element of such a pattern. But what about other elements of this pattern? How can we generalize the results [56],[72],[73]when the IR-cutoff tends to the UV-cutoff (formula (13))? What are modifications of the thermodynamic identity (23) in a high-energy deformed thermodynamics and how is it applied in high-energy (quantum) gravity? What are the aspects of using the Generalized Uncertainty Relations in Thermodynamics [12],[18],[58] (12)in this respect? It is clear that

these relations also form an element of high-energy thermodynamics.
By authors opinion, the methods developed to solve the problem of point (c) and elucidation of other above-mentioned problems may form the basis for a new approach to solution of the quantum gravity problem. And one of the keys to the **quantum gravity** problem is a better insight into the **high-energy thermodynamics**.

5 α–Representation of Einstein's Equations

Let us consider α-representation and high energy α-deformation of the Einstein's field equations for the specific cases of horizon spaces (the point (c) of Section 4). In so doing the results of the survey work ([74] p.p.41,42)are used. Then, specifically, for a static, spherically symmetric horizon in space-time described by the metric

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2 \tag{24}$$

the horizon location will be given by simple zero of the function $f(r)$, at $r = a$.
It is known that for horizon spaces one can introduce the temperature that can be identified with an analytic continuation to imaginary time. In the case under consideration ([74], eq.(116))

$$k_BT = \frac{\hbar c f'(a)}{4\pi}. \tag{25}$$

Therewith, the condition $f(a) = 0$ and $f'(a) \neq 0$ must be fulfilled.
Then at the horizon $r = a$ Einstein's field equations

$$\frac{c^4}{G}\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right] = 4\pi Pa^2 \tag{26}$$

may be written as the thermodynamic identity (23)([74] formula (119))

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_BT}\underbrace{\frac{c^3}{G\hbar}d\left(\frac{1}{4}4\pi a^2\right)}_{dS}\underbrace{-\frac{1}{2}\frac{c^4da}{G}}_{-dE} = \underbrace{Pd\left(\frac{4\pi}{3}a^3\right)}_{P\,dV} \tag{27}$$

where $P = T^r_r$ is the trace of the momentum-energy tensor and radial pressure. In the last equation da arises in the infinitesimal consideration of

Einstein's equations when studying two horizons distinguished by this infinitesimal quantity a and $a + da$ ([74] formula (118)).
Now we consider(27) in new notation expressing a in terms of the corresponding deformation parameter α. Then we have

$$a = l_{min}\alpha^{-1/2}. \tag{28}$$

Therefore,

$$f'(a) = -2l_{min}^{-1}\alpha^{3/2}f'(\alpha). \tag{29}$$

Substituting this into (26) or into (27), we obtain in the considered case of Einstein's equations in the "α–representation" the following:

$$\frac{c^4}{G}(-\alpha f'(\alpha) - \frac{1}{2}) = 4\pi P\alpha^{-1}l_{min}^2. \tag{30}$$

Multiplying the left- and right-hand sides of the last equation by α, we get

$$\frac{c^4}{G}(-\alpha^2 f'(\alpha) - \frac{1}{2}\alpha) = 4\pi P l_{min}^2. \tag{31}$$

But since usually $l_{min} \sim l_p$ (that is just the case if the Generalized Uncertainty Principle (GUP) is satisfied), we have $l_{min}^2 \sim l_p^2 = G\hbar/c^3$. When selecting a system of units, where $\hbar = c = 1$, we arrive at $l_{min} \sim l_p = \sqrt{G}$, and then (30) is of the form

$$-\alpha^2 f'(\alpha) - \frac{1}{2}\alpha = 4\pi P\vartheta^2 G^2, \tag{32}$$

where $\vartheta = l_{min}/l_p$. L.h.s. of (32) is dependent on α. Because of this, r.h.s. of (32) must be dependent on α as well, i. e. $P = P(\alpha)$.

Analysis of α-Representation of Einstein's Equations

Now let us get back to (27). In [74] the low-energy case has been considered, for which ([74] p.42 formula (120))

$$S = \frac{1}{4l_p^2}(4\pi a^2) = \frac{1}{4}\frac{A_H}{l_p^2}; \quad E = \frac{c^4}{2G}a = \frac{c^4}{G}\left(\frac{A_H}{16\pi}\right)^{1/2}, \tag{33}$$

where A_H is the horizon area. In our notation (33) may be rewritten as

$$S = \frac{1}{4}\pi\alpha^{-1}; \quad E = \frac{c^4}{2G}a = \frac{c^4}{G}\left(\frac{A_H}{16\pi}\right)^{1/2} = \frac{\vartheta}{2\sqrt{G}}\alpha^{1/2}. \tag{34}$$

We proceed to two entirely different cases: low energy (LE) case and high energy (HE) case. In our notation these are respectively given by

$$\text{A)}\alpha \to 0 \text{ (LE), B)}\alpha \to 1/4 \text{ (HE),}$$
$$\text{C)}\alpha \text{ complies with the familiar scales and energies.}$$

The case of C) is of no particular importance as it may be considered within the scope of the conventional General Relativity.
Indeed, in point A)$\alpha \to 0$ is not actually an exact limit as a real scale of the Universe (Infrared (IR)-cutoff $l_{max} \approx 10^{28} cm$), and then

$$\alpha_{min} \sim l_p^2/l_{max}^2 \approx 10^{-122}.$$

In this way A) is replaced by A1)$\alpha \to \alpha_{min}$. In any case at low energies the second term in the left-hand side (32) may be neglected in the infrared limit. Consequently, at low energies (32) is written as

$$-\alpha^2 f'(\alpha) = 4\pi P(\alpha)\vartheta^2 G^2. \tag{35}$$

Solution of the corresponding Einstein equation finding of the function $f(\alpha) = f[P(\alpha)]$ satisfying(35). In this case formulae (33) are valid as at low energies a semiclassical approximation is true. But from (35)it follows that

$$f(\alpha) = -4\pi\vartheta^2 G^2 \int \frac{P(\alpha)}{\alpha^2} d\alpha. \tag{36}$$

On the contrary, knowing $f(\alpha)$, we can obtain $P(\alpha) = T_r^r$.
But it is noteworthy that, when studying the infrared modified gravity [81],[82],[83], we have to make corrections for the considerations of point A1).

6 Possible High Energy α-Deformation of General Relativity

Let us consider the high-energy case B). Here two variants are possible.

I. First variant.
In this case it is assumed that in the high-energy (Ultraviolet (UV))limit the thermodynamic identity (27)(or that is the same (23)is retained but now all the quantities involved in this identity become α-deformed. This means that they appear in the α-representation with quantum corrections and are considered at high values of the parameter α, i.e. at α close to 1/4. In particular, the temperature T from equation (27) is changed by T_{GUP} (17), the entropy S from the same equation given by semiclassical formula (33) is changed by S_{GUP} (21), and so forth:

$$E \mapsto E_{GUP}, V \mapsto V_{GUP}.$$

Then the high-energy α-deformation of equation (27) takes the form

$$k_B T_{GUP}(\alpha) dS_{GUP}(\alpha) - dE_{GUP}(\alpha) = P(\alpha) dV_{GUP}(\alpha). \quad (37)$$

Substituting into (37) the corresponding quantities $T_{GUP}(\alpha), S_{GUP}(\alpha), E_{GUP}(\alpha), V_{GUP}(\alpha), P(\alpha)$ and expanding them into a Laurent series in terms of α, close to high values of α, specifically close to $\alpha = 1/4$, we can derive a solution for the high energy α-deformation of general relativity (37) as a function of $P(\alpha)$. As this takes place, provided at high energies the generalization of (27) to (37)is possible, we can have the high-energy α-deformation of the metric. Actually, as from (27) it follows that

$$f'(a) = \frac{4\pi k_B}{\hbar c} T = 4\pi k_B T \quad (38)$$

(considering that we have assumed $\hbar = c = 1$), we get

$$f'_{GUP}(a) = 4\pi k_B T_{GUP}(\alpha). \quad (39)$$

L.h.s. of (39) is directly obtained in the α-representation. This means that, when $f' \sim T$, we have $f'_{GUP} \sim T_{GUP}$ with the same factor of proportionality. In this case the function f_{GUP} determining the high-energy α-deformation of the spherically symmetric metric may be in fact derived by the expansion of T_{GUP}, that is known from (17), into a Laurent series in terms of α close to high values of α (specifically close to $\alpha = 1/4$), and by the subsequent integration.
It might be well to remark on the following.

6.1 As on going to high energies we use (GUP), ϑ from equation (32)is expressed in terms of α'–dimensionless constant from GUP (1),(17):$\vartheta = 2\alpha'$.

6.2 Of course, in all the formulae including l_p this quantity must be changed by $G^{1/2}$ and hence l_{min} by $\vartheta G^{1/2} = 2\alpha' G^{1/2}$.

6.3 As noted in the end of subsection 6.1, and in this case also knowing all the high-energy deformed quantities $T_{GUP}(\alpha), S_{GUP}(\alpha), E_{GUP}(\alpha), V_{GUP}(\alpha)$, we can find $P(\alpha)$ at α close to 1/4.

6.4 Here it is implicitly understood that the Ultraviolet limit of Einstein's equations is independent of the starting horizon space. This assumption is quite reasonable. Because of this, we use the well-known formulae for the modification of thermodynamics and statistical mechanics of black holes in the presence of GUP [26]–[32].

6.5 The use of the thermodynamic identity (37) for the description of the high energy deformation in General Relativity implies that on going to the UV-limit of Einsteins equations for horizon spaces in the thermodynamic representation (consideration) we are trying to remain within the scope of **equilibrium statistical mechanics** [75] (**equilibrium thermodynamics**) [76]. However, such an assumption seems to be too strong. But some grounds to think so may be found as well. Among other things, of interest is the result from [26] that GUP may prevent black holes from their total evaporation. In this case the Plancks remnants of black holes will be stable, and when they are considered, in some approximation the **equilibrium thermodynamics** should be valid. At the same time, by authors opinion these arguments are rather weak to think that the quantum gravitational effects in this context have been described only within the scope of **equilibrium thermodynamics**[76].

II. Second variant.
According to the remark of **6.5**, it is assumed that the interpretation of Einstein's equations as a thermodynamic identity (27) is not retained on going to high energies (UV–limit), i.e. at $\alpha \rightarrow 1/4$, and the situation is adequately described exclusively by **non-equilibrium thermodynamics**[76],[77]. Naturally, the question arises: which of the additional terms introduced in (27) at high energies may be leading to such a description?
In the [41],[42] it has been shown that in case the cosmological term Λ is a dynamic quantity, it is small at low energies and may be sufficiently large at high energies. In the right-hand side of (32) in the α–representation the additional term $GF(\Lambda(\alpha))$ is introduced:

$$-\alpha^2 f'(\alpha) - \frac{1}{2}\alpha = 4\pi P\vartheta^2 G^2 - GF(\Lambda(\alpha)), \tag{40}$$

where in terms of $F(\Lambda(\alpha))$ we denote the term including $\Lambda(\alpha)$ as a factor. Then its inclusion in the low-energy case (26)(or in the α -representation (32)) has actually no effect on the thermodynamic identity (27)validity, and

consideration within the scope of equilibrium thermodynamics still holds true. It is well known that this is not the case at high energies as the Λ-term may contribute significantly to make the "process" non-equilibrium in the end [76],[77].
Is this the only cause for violation of the thermodynamic identity (27) as an interpretation of the high-energy generalization of Einstein's equations? Further investigations are required to answer this question.

7 Conclusion

This work presents the first steps to incorporation of the deformation parameters of a quantum field theory at Plancks scales into the high-energy deformation of General Relativity (GR). Further, the corresponding calculations should follow with an adequate interpretation. It is interesting to consider the high energy α-deformation of GR in a more general case. The problem is how far a thermodynamic interpretation of Einstein's equations may be extended? We should remember that, as in all the deformations considered a minimal length at the Planck level $l_{min} \sim l_p$ has been involved, a minimal volume should also be the case $V_{min} \sim V_p = l_p^3$, and this is of particular importance for high energy thermodynamics (some indications to this fact have been demonstrated in [41],[42]).
Besides, in this paper we have treated QFT with a minimal length, i.e. with the UV-cutoff. Consideration of QFT with a minimal momentum (or IR-cutoff) [32] necessitates an adequate extension of α-deformation in QFT with the introduction of new parameters significant in the IR-limit.
It seems that some hints to a nature of such deformation may be found from the works devoted to the infrared modification of gravity [81]–[83].

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