The Quantum Primordial Black Holes,Dimensionless Small Parameter,Inflationary Cosmology and Non-Gaussianity

Alexander Shalyt-Margolin [1]

Research Institute for Nuclear Problems,Belarusian State University, 11 Bobruiskaya str., Minsk 220040, Belarus

Abstract

In the present work consideration is given to the primordial black holes (**pbhs**) in the Schwarzschild-de Sitter Metric with small mass (ultralight) in the preinflationary epoch. Within the scope of natural assumptions, it has been shown that the quantum-gravitational corrections (**qgcs**) to the characteristics of such black holes can contribute to all the cosmological parameters, shifting them compared with the semiclassical consideration. These contributions are determined by a series expansion in terms of a small parameter dependent on the hole mass (radius). For this pattern different cases have been considered (stationary, black hole evaporation...). It has been demonstrated that involvement of (**qgcs**) leads to a higher probability for the occurrence of such **pbhs**. Besides, high-energy deformations of Friedmann Equations created on the basis of these corrections have been derived for different patterns. In the last section of this work it is introduced a study into the contributions generated by the abovementioned **qgcs** in inflationary cosmological perturbations. Besides, it has been shown that non-Gaussianity of these perturbations is higher as compared to the semi-classical pattern.

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[1]E-mail: a.shalyt@mail.ru; alexm@hep.by

1 Introduction

The primordial black holes (**pbhs**) [1]–[3] in the Early Universe are due to gravitational collapse of the high-density matter [4]. In [5]–[7] a sufficiently accurate estimate of the mass **pbhs** $M(t_M)$ formed during the period of time t since the Big Bang has been obtained

$$M(t_M) \approx \frac{c^3 t_M}{G} \approx 10^{15} \left(\frac{t}{10^{-23}\ \mathrm{s}} \right) g. \tag{1}$$

As seen, for small times close to the Planckian time $t_M = t_p \approx 10^{-43}$s, the mass of **pbhs** is close to the Planck mass $M(t_M) \approx 10^{-5} g$. The names of such black holes were varying with time: "mini-black holes","micro-black holes", and, e.g. in [8], they were referred to as "ultralight primordial black holes". The author of this paper uses for such **pbhs** the name **quantum pbhs** (or **qpbhs**) introduced in [9],[10] and notes that quantum-gravitational effects for these objects could be significant. Of particular interest are **pbhs** arising in the preinflationary epoch. In [11] a semiclassical approximation was used to study the problem of scalar perturbations due to such **pbhs**. But, considering that all the processes in this case proceed at very high energies E close to the Planckian $E \simeq E_p$, the inclusion of quantum-gravity corrections **qgcs** for these black holes in this pattern is necessary if quantum gravity exists [12]. Despite the fact that presently there is no self-consistent theory of quantum gravity, a consensus is reached on correctness of some approaches to the theory, specifically, replacement of the Heisenberg Uncertainty Principle **(HUP)** by the Generalized Uncertainty Principle **(GUP)** on going to high (Planck's) energies, used in this paper.

Within the scope of a natural assumption based on the results from [11], in the present work the author studies the problem, how the above-mentioned **qgcs** for quantum **pbhs** can shift the inflationary parameters and contribute to cosmological perturbations involved in the inflationary process. Moreover, it is shown that inclusion of **qgcs**:

a) increases the probability of the generation of **pbhs**;

b) leads to the enhancement of non-Gaussianity in cosmological perturbations.

In what follows the abbreviation **qgcs** is associated with the foregoing quantum-gravity corrections. Beginning a study of **qgcs** in [**?**], here the author, using the definitions from [**?**], presents much more general results referring to the emergence probability of quantum **pbhs** as well as of quantum fluctuations,perturbations and non-Gaussianity.
The paper is structured as follows.
Section 2 presents the instruments used to obtain the principal results. Section 3 shows how **qgcs** shift the inflationary parameters within a natural assumption from [11]. In Section 4 it is demonstrated that inclusion of **qgcs** increases the occurrence probability for such **pbhs**. In Section 5 the high-energy deformations of Friedmann Equations on the basis of **qgcs** are derived for different cases. Finally, Section 6 begins a study of the contributions made by **qgcs** into different cosmological perturbations under inflation and, due to the involvement of **qgcs**, demonstrates the enhancement of non-Gaussianity for different perturbations revealed as growing moduli of **bispectrums**.
In what follows the normalization $c = \hbar = 1$ is used, for which we have $G = l_p^2$.

2 PBHs with the Schwarzschild-de Sitter Metric in the Early Universe

It should be noted that Schwarzschild black holes in real physics (cosmology, astrophysics) are idealized objects. As noted in (p.324,[13]): "Spherically symmetric accretion onto a Schwarzschild black hole is probably only of academic interest as a testing for theoretical ideas. It is of little relevance for interpretations of the observations data. More realistic is the situation where a black hole moves with respect to the interstellar gas..."
Nevertheless, black holes just of this type may arise and may be realistic in the early Universe. In this case they are **pbhs**.
During studies of the early Universe for such **pbhs** the Schwarzschild metric [14],[13]

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (2)$$

for **pbhs** is replaced by the Schwarzschild-de Sitter (SdS) metric [11] that is associated with Schwarzschild black holes with small mass M in the early Universe, in particular in pre-inflation epoch

$$ds^2 = -f(\tilde{r})dt^2 + \frac{d\tilde{r}^2}{f(\tilde{r})} + \tilde{r}^2 d\Omega^2 \tag{3}$$

where $f(\tilde{r}) = 1 - 2GM/\tilde{r} - \Lambda\tilde{r}^2/3 = 1 - 2GM/\tilde{r} - \tilde{r}^2/L^2, L = \sqrt{3/\Lambda} = H_0^{-1}$, M - black hole mass, Λ – cosmological constant, and $L = H_0^{-1}$ is the Hubble radius.

In general, such a black hole may have two different horizons corresponding to two different zeros $f(\tilde{r})$: event horizon of a black hole and cosmological horizon. This is just so in the case under study when a value of M is small [15],[16]. In the general case of $L \gg GM$, for the event horizon radius of a black hole having the metric (3), $r_{\rm H}$ takes the following form (formula (9) in [17]):

$$r_{\rm H} \simeq 2GM\left[1 + \left(\frac{r_M}{L}\right)^2\right], where\ r_M = 2MG. \tag{4}$$

Then, due to the assumption concerning the initial smallness of Λ, we have $L \gg r_M$. In this case, to a high accuracy, the condition $r_{\rm H} = r_M$ is fulfilled, i.e. for the considered (SdS) BH we can use the formulae for a Schwarzschild BH, to a great accuracy.

Remark 2.1.

Note that, because Λ is very small, the condition $L \gg GM$ and hence the formula of (4) are obviously valid not only for black hole with the mass $M \propto m_p$ but also for a much greater range of masses, i.e. for black holes with the mass $M \gg m_p$, taking into account the condition $L \gg GM$. In fact we obtain ordinary Schwarzschild black holes (2) with small radius.

Specifically, for the energies on the order of Plank energies (quantum gravity scales) $E \simeq E_p$, the Heisenberg Uncertainty Principle (**HUP**) [18]

$$(\delta X)\,(\delta P) \geq \frac{\hbar}{2}, \tag{5}$$

may be replaced by the Generalized Uncertainty Principle (**GUP**) [19]

$$(\delta X)\,(\delta P) \geq \frac{\hbar}{2}\left\langle \exp\left(\frac{\alpha^2 l_p^2}{\hbar^2}P^2\right)\right\rangle. \tag{6}$$

Then there is a possibility for existence of Planck Schwarzschild black hole, and accordingly of a Schwarzschild sphere (further referred to as "minimal") with the minimal mass M_0 and the minimal radius r_{min} (formula (20) in [19]) that is a theoretical minimal length r_{min}:

$$r_{min} = l_{min} = (\delta X)_0 = \sqrt{\frac{e}{2}}\alpha l_p, \quad M_0 = \frac{\alpha\sqrt{e}}{2\sqrt{2}}m_p, \tag{7}$$

where α - model-dependent parameters on the order of 1, e - base of natural logarithms, and $r_{min} \propto l_p, M_0 \propto m_p$.
In this case, due to GUP (6), the physics becomes nonlocal and the position of any point is determined accurate to l_{min}. It is impossible to ignore this nonlocality at the energies close to the Planck energy $E \approx E_p$, i.e. at the scales $l \propto l_p$ (equivalently we have $l \propto r_{min} = l_{min}$).
Actually, [19] presents calculated values of the mass M and the radius R for Schwarzschild BH with regard to the quantum-gravitational corrections within the scope of GUP (6).
With the use of the normalization $G = l_p^2$ adopted in [19], temperature of a Schwarzschild black hole having the mass M (the radius R) [13] in a semi-classical approximation takes the form

$$T_H = \frac{1}{8\pi GM}. \tag{8}$$

Within the scope of GUP (6),the temperature T_H with regard to (**qgc**) is of the form ((23) and (25) in [19]))

$$T_{H,q} = \frac{1}{8\pi MG}\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = \frac{1}{8\pi MG}\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{A_0}{A}\right)\right)\right) =$$
$$= \frac{1}{8\pi MG}\left(1+\frac{1}{2e}\left(\frac{M_0}{M}\right)^2+\frac{5}{8e^2}\left(\frac{M_0}{M}\right)^4+\frac{49}{48e^3}\left(\frac{M_0}{M}\right)^6+\ldots\right), \tag{9}$$

where A is the black hole horizon area of the given black hole with mass M and event horizon r_M ,$A_0 = 4\pi(\delta X)_0^2$ is the black hole horizon area of a minimal quantum black hole from formula (7) and $W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right) =$

$W\left(-\frac{1}{e}\left(\frac{A_0}{A}\right)\right)$ – value at the corresponding point of the Lambert W-function $W(u)$ satisfying the equation (formulae (1.5) in [20] and (9) in [19])

$$W\left(u\right)e^{W(u)} = u. \tag{10}$$

$W\left(u\right)$ is the multifunction for complex variable $u = x+yi$. However, for real $u = x, -1/e \leq u < 0, W\left(u\right)$ is the single-valued continuous function having two branches denoted by $W_0(u)$ and $W_{-1}(u)$, and for real $u = x, u \geq 0$ there is only one branch $W_0(u)$ [20].

Obviously, the quantum-gravitational correction (**qgc**) (9) presents a *deformation* (or more exactly, the *quantum deformation* of a classical black-holes theory from the viewpoint of the paper [21] with the deformation parameter A_0/A):

$$\frac{A_0}{A} = \frac{4\pi r_h^2}{4\pi r_M^2} = \frac{l_{min}^2}{r_M^2}, \tag{11}$$

where $r_h = l_{min}$ is the horizon radius of minimal **pbh** from formula (7).

It should be noted that this deformation parameter

$$l_{min}^2/r_M^2 \doteq \alpha_{r_M} \tag{12}$$

has been introduced by the author in his earlier works [22],[23], where he studied deformation of quantum mechanics at Planck scales in terms of the deformed quantum mechanical density matrix. In the Schwarzschild black hole case $\alpha_{r_M} = l_{min}^2\mathcal{K}$ – Gaussian curvature $\mathcal{K} = 1/\alpha_{r_M}$ of the black-hole event horizon surface [24].

It is clear that, for a great black hole having large mass M and great event horizon area A, the deformation parameter $\frac{1}{e}\left(\frac{M_0}{M}\right)^2$ is vanishingly small and close to zero. Then a value of $W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)$ Is also close to $W(0)$. As seen, $W(0) = 0$ is an obvious solution for the equation (10). We have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) \approx 1. \tag{13}$$

So, a black hole with great mass $M \gg m_p$ necessitates no consideration of **qgcs**.

But in the case of small black holes we have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > 1. \tag{14}$$

In formulae above it is assumed that $M > M_0$, i.e. the black hole under study is not minimal (7).
We can rewrite the formula of (9) as follows:

$$T_{H,q} = \frac{1}{8\pi M_q G}, M_q = M\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right);$$
$$R_q = 2M_q G = R\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right), \tag{15}$$

where M_q and R_q are respectively the initial black-hole mass and event horizon radius considering **qgcs** caused by GUP (6).
Note that in a similar way M_q is involved in ([19] formula (26)) as a function of the black-hole temperature. But, instead of the small parameter M_0/M, the author uses the small parameter T_H/T_H^{max}, where T_H^{max} is the black hole maximum temperature.
Remark 2.2
It is clear that the formula (15) with the substitution of $M \mapsto M_q$ is of the same form as formula (8), in fact representing (9),i.e. in the formula for temperature of a black hole the inclusion of **qgcs** *may be realized in two ways with the same result: (a)the initial mass M remains unaltered and* **qgcs** *are involved only in the formula for temperature, in this case (9); (b)***qgcs** *are involved in the mass-the above-mentioned substitution takes place $M \mapsto M_q$ (formula(15)). Such "duality" is absolutely right in this case if a black hole is considered in the stationary state in the absence of accretion and radiation processes. Just this case is also studied in the paper.*
A recent preprint [25] in the case (b) for the space-time dimension $D \geq 4$, using approaches to quantum gravity of the alternative GUP, gives a formula for the mass M_q of a black hole with a due regard to **qgc**

$$M_q = \left[1 - \eta\exp\left(-\frac{\pi r_0^{D-2}}{G_D}\right)\right]^{D-3} M. \tag{16}$$

Here in terms of [25] r_0 is the Schwarzschild radius of the primordial black hole with the mass M,G_D-gravitational constant in the dimension D, and $\eta = [0,1]$ is a parameter. In case under study this parameter, as distinct from cosmology, has no relation to conformal time. Obviously, for $\eta = 0$ we have a semiclassical approximation and, as noted in [25], the case when $\eta = 1$ corresponds to **qgc** *as predicted by a string theory.*
Remark 2.3
It should be noted that, with the expansion $\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$ *in terms of the small parameter* $\alpha_{r_M} = (M_0/M)^2$*, in formula (9) one can easily obtain the small-parameter expansion of the inverse number* $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$*, and also of all the integer powers for this exponent, specifically for its square* $\exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$.

3 Inflation Parameters Shifts Generated by QGCs

To this end in cosmology, in particular inflationary, the metric (3) is conveniently described in terms of the conformal time η [11]:

$$ds^2 = a^2(\eta)\left\{ - d\eta^2 + \left(1 + \frac{\mu^3\eta^3}{r^3}\right)^{4/3}\left[\left(\frac{1 - \mu^3\eta^3/r^3}{1 + \mu^3\eta^3/r^3}\right)^2 dr^2 + r^2 d\Omega^2\right]\right\}, \tag{17}$$

where $\mu = (GMH_0/2)^{1/3}$, H_0 – de Sitter-Hubble parameter and scale factor, a – conformal time function η:

$$a(\eta) = -1/(H_0\eta), \eta < 0. \tag{18}$$

Here r satisfies the condition $r_0 < r < \infty$ and a value of $r_0 = -\mu\eta$ in the reference frame of (17) conforms to singularity of the back hole.
Due to (4), μ may be given as

$$\mu = (r_M H_0/4)^{1/3}, \tag{19}$$

where r_M is the radius of a black hole with the SdS Schwarzschild-de Sitter metric (3).
Remark 3.1.

As we consider quantum **pbhs** *with the SdS-metric, in this case, as noted in (4), they to a high accuracy are coincident with the Schwarzschild black holes (2) and hence they have the identical formulae for* **qgcs***:(15),(16)...*
Further we consider the contribution made by these **qgcs** *into the quantities associated with inflation: inflationary parameters, cosmological perturbations etc. It should be made clear:*
In [11] in general only the case $\mu = const$ *is considered and, as noted in [11], for the case* $\mu \neq const$ *we can use only the pattern including the radiation processes of* **pbhs**. *However, the value of* $\mu = const$ *itself contains no information concerning the treatment of either the initial quantum* **pbh** *in a semiclassical approximation or the consideration with regard to* **qgcs**. *Obviously, in this case involvement of* **qgcs** *shifts all the parameters derived for a semiclassical (canonical) pattern.*
Let us consider the following pattern related to that studied in [11]: it is supposed that, as the mass M of **pbh** may be changed due to the radiation process, the corresponding change takes place for μ – in the general case we have ($\mu \neq const$) in view of these processes. But further it is assumed that after termination of these processes μ is unaltered with regard to **qgcs**, i.e. in formula (19) we have $\mu = (r_M H_0/4)^{1/3} = (r_{M_q} H_{0,q}/4)^{1/3}$, where $r_{M_q}, H_{0,q}$ - values of r_M, H_0, respectively, with due regard for **qgcs**.
3.1. *The Stationary picture.*
From the start of creation, the primordial black hole, with the mass M and the event horizon area A, is considered in the absence of absorption and radiation processes. It may be assumed that such quantum **pbh** was generated immediately before the onset of inflation, when there were no absorption and radiation processes. On the other hand, **3.1** is completely consistent with the paradigm in [9], [10] presuming the absence of Hawking radiation for quantum **pbh**, with the mass and the event horizon radius close to the Planckian values $M \approx m_p, r_M \approx l_p$.
As $\mu = const$ and **pbh** we consider in the stationary state, then, due to **Remark 2.2** with regard for **qgcs**, replacement $r_M \mapsto r_{M_q}$ in this formula leads to replacement of $H_0 \to H_{0,q}$, due to **Remark 3.1.** meeting the condition
$$\mu = (r_M H_0/4)^{1/3} = (r_{M_q} H_{0,q}/4)^{1/3}. \qquad (20)$$

Here $r_{M_q} = R_q$ from the general formula (15).
Based on the last formula and formulae (9),(12),(15) it directly follows that

$$H_{0,q} = H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right). \quad (21)$$

Then **qgc** for the scale factor $a(\eta)$ (18)

$$a(\eta) \to a(\eta)_q \doteq -1/(H_{0,q}\eta) = -1/(H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\eta) =$$

$$= -1/(H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\eta) = a(\eta)\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right), \eta < 0, \quad (22)$$

and for the Hubble parameter in general case $H(\eta) \doteq H$

$$H = a'(\eta)/a^2(\eta) \mapsto H_q(\eta) = a'(\eta)_q/a^2(\eta)_q \quad (23)$$

As directly follows from the last formula, in this pattern H_0 and H are identically transformed, i.e. we have

$$H \mapsto H_q = H \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) \quad (24)$$

Because the potential energy of inflation V is related to the Hubble parameter H by the Friedmann equation (formula (12.12) in [28]) $H^2 = 8\pi V/(3M_p^2)$,from (21) we can derive a "shift" for V that is due to quantum-gravitational corrections for the primordial Schwarzschild black hole with the mass M as follows:

$$[V = 3M_p^2H^2/(8\pi)] \mapsto V_q = 3M_p^2H_q^2/(8\pi) =$$

$$= 3\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)M_p^2H^2/(8\pi) = \exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)V, \quad (25)$$

In a similar way, taking account of **qgcs** for quantum **pbhs** (formulae (9,(15), we can find these "shifts" for all inflationary parameters, in particular

$$(a \sim H^{-1}) \mapsto a_q \sim H_q^{-1};$$
$$(V \sim H^2) \mapsto V_q \sim H_q^2;$$
$$(\dot{\phi} = -\frac{V'(\phi)}{3H} \sim H) \mapsto \dot{\phi}_q = (-\frac{V'(\phi)}{3H})_q \sim H_q;$$
$$(\dot{H} = \frac{1}{2m_p}(\frac{8\pi}{3V})^{1/2} V'(\phi)\dot{\phi} \sim H^2) \mapsto \dot{H}_q = \frac{1}{2m_p}[(\frac{8\pi}{3V})^{1/2} V'(\phi)\dot{\phi}]_q \sim H_q^2;$$
$$(\ddot{\phi} = -\frac{m_p^2}{8\pi}(\frac{V''}{V} - \frac{1}{2}(\frac{V'}{V})^2)H\dot{\phi} \sim H^2) \mapsto \ddot{\phi}_q \sim H_q^2, ...(26)$$

with retention of some part of them on the transformation $H \mapsto H_q$,for example it is easy to check retention of the deceleration parameters $\epsilon, \widetilde{\eta}$ [28]:

$$\epsilon = -\frac{\dot{H}}{H^2} = m_p \frac{V'^2}{V^2} = \epsilon_q = -\frac{\dot{H}_q}{H_q^2} = m_p \frac{V_q'^2}{V_q^2},$$
$$\widetilde{\eta} = \frac{m_p^2}{8\pi}\frac{V''}{V} = \widetilde{\eta}_q. \qquad (27)$$

Here in the last two formulae (26),(27) a point means differentiation with respect to t, but a prime means differentiation with respect to the field ϕ. Let us denote the second deceleration parameter $\widetilde{\eta}$ instead of η [28], to avoid confusion with the conformal time.
As we have $\epsilon = \epsilon_q, \widetilde{\eta} = \widetilde{\eta}_q$,the slow-roll conditions (formula (27) in [28]) in the inflationary scenario with regard to **qgcs** for **qpbhs** remains unaltered. *Due to formula (9), all the above-mentioned shifts of the inflationary parameters generated by* **qgcs** *for quantum* **pbhs** *in the pre-inflationary period may be series expanded in terms of the small parameter* $(M_0/M)^2$ *(same* α_{r_M}*).*

3.2 *Black Hole Evaporation and* **qgcs**
(This case studied in [?] is given for completeness of the presentation.)
Also,black holes are associated with the process of Hawking radiation (evaporation). The primordial black holes are no exception. In the general case

this process is considered only within the scope of a semiclassical approximation (without consideration of the quantum-gravitational effects). Because of this, it is assumed that a primordial black hole may be completely evaporated [13].
Still, in this pattern the situation is impossible due to the validity of GUP (6) and due to the formation of a minimal (nonvanishing) Planckian remnant as a result of evaporation (7) [29],[19].
We can compare the mass loss for a black hole in this process when using a semiclassical approximation and with due regard for **qgcs**.
Let M be the mass of a primordial black hole. Then a loss of mass as a result of evaporation, according to the general formulae, takes the following form ([13],p.356):

$$\frac{dM}{dt} \sim \sigma T_{\rm H}^{4} A_{M}, \tag{28}$$

where $T_{\rm H}$ - temperature of a black hole with the mass M,A_M - surface area of the event horizon of this hole $A_M = 4\pi r_M^2$, and $\sigma = \pi^2 k^4/(60\hbar^3 c^2)$ is the Stefan-Boltzmann constant.
Using this formula for the same black hole but with regard to **qgcs**, we can get the mass loss $[dM/dt]_q$ in this case

$$[\frac{dM}{dt}]_q \sim \sigma T_{{\rm H},q}^{4} A_{M}, \tag{29}$$

where $T_{{\rm H},q}$ - temperature of a black hole with the same mass M, when taking into consideration **qgcs** (9).
For all the foregoing formulae associated with a random black hole having the mass M, the following estimate is correct ((10.1.19) in [13]):

$$-\frac{dM}{dt} \sim b(\frac{M_p}{M})^2(\frac{M_p}{t_p})^2 N, \tag{30}$$

where $b \approx 2.59 \times 10^{-6}$, and N is the number of the states and species of particles that are radiated. The minus sign in the left part of the last formula denotes that the mass of a black hole diminishes as a result of evaporation, i.e. we have $dM/dt < 0$.
Unfortunately, the last formula is hardly constructive as it is difficult to estimate the number N, especially at high energies $E \simeq E_p$.

Nevertheless, using the terminology and symbols of this paper, and also the results from [19], the formula (30) for the mass loss by a black hole with regard to **qgcs** may be written in a more precise and constructive form. Really, according to formula (45) in [19], within the scope of GUP (6) we will have

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4} \exp\left(-2W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) \times$$
$$\times \left(1 - \frac{8\gamma_2}{e\gamma_1}\left(\frac{M_0}{M}\right)^2 \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\right), \qquad (31)$$

where $\gamma_1 = \frac{\pi^2}{480}$, $\gamma_2 = \frac{\pi^2}{16128}$.
The minus sign in the right side of the last formula means the same as the minus sign in the left side of formula (30).
Due to (12), formula (31) is of the following form:

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4} \exp(-2W\left(-\frac{1}{e}\alpha_{r_M}\right)) \times$$
$$\times \left(1 - \frac{8\gamma_2}{e\gamma_1}\alpha_{r_M} \exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\right). \qquad (32)$$

We can expand the right sides of formulae (31) and (32) into a series in terms of the small parameter $e^{-1}(M_0/M)^2 = e^{-1}\alpha_{r_M}$ (formula (46) in [19]) that, proceeding from the deformation parameter $\alpha_{r(M)}$, takes the form

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4}\left(1 + \frac{2}{e}\alpha_{r_M} + \frac{4}{e^2}\left(1 - \frac{2\gamma_2}{e\gamma_1}\right)\alpha_{r_M}^2 + \frac{25}{3e^3}\left(1 - \frac{72\gamma_2}{25e\gamma_1}\right)\alpha_{r_M}^3 + \ldots\right). \qquad (33)$$

Neglecting the last equation due to the time interval chosen, e.g., due to $\triangle t = t_{infl} - t_M$,where t_{infl}–time of the inflation onset and t_M– time during which the black hole under study has been formed, formula (1), the mass loss for a black hole with regard to **qgcs** by the inflation onset time may be given as

$$\Delta_{Evap,q}M(t_M, t_{infl}) \doteq \int\limits_{t_M}^{t_{infl}} \frac{dM}{dt} =$$

$$= -\int\limits_{t_M}^{t_{infl}} \frac{\gamma_1}{M^2 l_p^4}\left(1+\frac{2}{e}\alpha_{r_M}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{r_M}^2+\frac{25}{3e^3}\left(1-\frac{72\gamma_2}{25e\gamma_1}\right)\alpha_{r_M}^3+\ldots\right). \quad (34)$$

Next, we can determine the mass of a black hole after its evaporation until the inflation onset with regard to **qgcs**

$$M_{Evap,q}(t_{M_q}, t_{infl}) \doteq M + \Delta_{Evap,q}M_q(t_{M_q}, t_{infl}). \quad (35)$$

In the pattern of a semiclassical approximation the above-mentioned formulae are greatly simplified because in this case $\alpha_{r_M} = 0$ due to the absence of a minimal black hole.
Then in a semiclassical pattern formula (35), with the use of the suggested formalism, takes the following form:

$$M_{Evap}(t_M, t_{infl}) \doteq M + \Delta_{Evap}M(t_M, t_{infl}), \quad (36)$$

where

$$\Delta_{Evap}M(t_M, t_{infl}) = \int\limits_{t_M}^{t_{infl}} \frac{dM}{dt} = -\int\limits_{t_M}^{t_{infl}} \frac{\gamma_1}{M^2 l_p^4}. \quad (37)$$

Accordingly, for the radii $M_{Evap}(t_M, t_{infl}), M_{Evap,q}$ we can get

$$\begin{aligned} r(M_{Evap}) &= 2GM_{Evap}(t_M, t_{infl}),\\ r(M_{Evap,q}) &= 2GM_{Evap,q}(t_M, t_{infl}). \end{aligned} \quad (38)$$

In accordance with **Remark 3.1**, we have

$$\mu_{Evap} \doteq (r_{M_{Evap}} H_{0,Evap}/4)^{1/3} = (r_{M_{Evap,q}} H_{0,Evap,q}/4)^{1/3};$$
$$H_{0,Evap,q} = \frac{r_{M_{Evap}}}{r_{M_{Evap,q}}} H_{0,Evap}. \quad (39)$$

The right side of the last line in formula (39) gives the **"quantum-gravitational shifts"** (abbreviated as **qgs**) of the de Sitter Hubble parameter H_0 for black holes evaporation process.
Substituting $H_{0,Evap,q}$ from (39) into formulae (25)–(27) and so on, we can obtain **qgsc** for all cosmological parameters in the inflationary scenario when a primordial black hole evaporates before the inflation onset.
Remark 3.2.
By the present approach we can consider the case of the particle absorption by a **pbh**. Let the Schwarzschild-de Sitter **pbh** of the mass M has the event horizon area A.In [26],[27] "a minimal increment" of the event horizon area for the black hole absorbing a particle with the energy E and with the size R:$(\Delta A)_0 \simeq 4l_p^2 (\ln 2)\, ER$ has been estimated within the scope of the Heisenberg Uncertainty Principle **HUP**. In quantum consideration we have $R \sim 2\delta X$ and $E \sim \delta P$. Within the scope of GUP this " minimal increment" is replaced by $(\Delta A)_{0,q}$ as follows (formula (27) in [19]):

$$(\Delta A)_{0,q} \approx 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right) = 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) \quad (40)$$

Assuming that an arbitrary increment of the event horizon area A (same with the mass M) may be represented as a chain of "minimal increments" (for quantum **pbhs** with the mass close to that of the Planck's such an assumption is fairly justified) for $\mu = const$ and any absorption we can compare in the given approach the values of all cosmological parameters in the semiclassical approximation and their "shifts" generated by **qgcs**.

4 Quantum-Gravity Corrections for Appearance Probabilities PBHs in the Pre-Inflationary Era

There is the problem of estimating the probability of occurrence for **pbh** with Schwarzschild-de Sitter **SdS** metric (3) in the pre-inflation epoch.

This problem has been studied in [11] without due regard for **qgcs**. Let us demonstrate that consideration of **qgcs** in this case makes the probability of arising such **pbhs** higher.
Similar to [11], it is assumed that in pre-inflation period non-relativistic particles with the mass $m < M_p$ are dominant (Section 3 in [11]). For convenience, let us denote the Schwarzschild radius r_M by R_S.
When denoting, in analogy with [11], by $N(R,t)$ the number of particles in a *comoving* ball with the physical radius $R = R(t)$ and the volume V_R at time t, in the case under study this number (formula (3.9) in [11]) will have by **qgc**:

$$N(R,t) \mapsto N(R,t)_q;$$
$$(\langle N(R,t)\rangle = \frac{m_p^2 H^2 R^3}{2m}) \mapsto (\langle N(R,t)_q\rangle = \frac{m_p^2 H_q^2 R^3}{2m}). \tag{41}$$

Here the first part of the last formula agrees with formula (3.9) in [11], whereas H, H_q in this case are in agreement with formulae (23),(24). And from (24) it follows that

$$\langle N(R,t)_q\rangle = \langle N(R,t)\rangle \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \tag{42}$$

According to (15), it is necessary to replace the Schwarzschild radius R_S by $R_{S,q} = R_S \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$.
Then from the general formula $N(R_S,t) = \langle N(R_S,t)\rangle + \delta N(R_S,t)$, used because of the replacement of $R_S \mapsto R_{S,q}$, we obtain an analog of (3.12) from [11]

$$\delta N > \delta N_{\rm cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S,t)_q\rangle = \frac{m_p^2 R_{S,q}}{2m}[1-(HR_S)^2] =$$
$$= \frac{m_p^2 R_S}{2m}[1-(HR_S)^2]\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = \delta N_{\rm cr}\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \tag{43}$$

In the last formula in square brackets we should have $(H_q R_{S,q})^2$ instead of $(HR_S)^2$ but, as we consider the case $\mu = const$, these quantities are

coincident.
It should be noted that here the following condition is used:

$$HR_S < 1, \tag{44}$$

i.e. Schwarzschild radius R_S less than Hubble radius, $R_S < R_H = 1/H$.
As we have $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) < 1$, then

$$\delta N_{\rm cr,q} < \delta N_{\rm cr}. \tag{45}$$

Considering that for the formation of a Schwarzschild black hole with the radius R_S it is required that, due to statistical fluctuations, the number of particles $N(R_S, t)$ with the mass m within the black hole volume $V_{R_S} = 4/3\pi R_S^3$ be in agreement with the condition [11]

$$N(R_S, t) > R_S M_p^2/(2m), \tag{46}$$

which, according to **qgc** in the formula of (15), may be replaced by

$$N(R_{S,q}, t) > R_{S,q} M_p^2/(2m) = \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) R_S M_p^2/(2m). \tag{47}$$

As follows from these expressions, with regard to **qgc** for the formation of **pbh** in the pre-inflation period, the number of the corresponding particles may be lower than for a black hole without such regard, leading to a higher probability of the formation.
Such a conclusion may be made by comparison of this probability in a semi-classical consideration (formula (3.13) in [11])

$$P\big(\delta N(R_S, t) > \delta N_{\rm cr}(R_S, t)\big) = \int_{\delta N_{\rm cr}}^{\infty} d(\delta N) P(\delta N) \tag{48}$$

and with due regard for **qgc**

$$P\big(\delta N(R_{S,q}, t) > \delta N_{\rm cr}(R_{S,q}, t)\big) = \int_{\delta N_{\rm cr,q}}^{\infty} d(\delta N) P(\delta N). \tag{49}$$

Considering that in the last two integrals the integrands take positive values and are the same, whereas the integration domain in the second integral is wider due to (45), we have

$$\int_{\delta N_{\mathrm{cr,q}}}^{\infty} d(\delta N)P(\delta N) =$$

$$= \int_{\delta N_{\mathrm{cr,q}}}^{\delta N_{\mathrm{cr}}} d(\delta N)P(\delta N) + \int_{\delta N_{\mathrm{cr}}}^{\infty} d(\delta N)P(\delta N) > \int_{\delta N_{\mathrm{cr}}}^{\infty} d(\delta N)P(\delta N). \quad (50)$$

As follows from the last three formulae, in the case under study the probability that the above-mentioned **pbh** will be formed is higher with due regard for **qgc**.

5 High Energy Deformations of Friedmann Equations

Based on the obtained results, it is inferred that there is the deformation (having a quantum-gravitational character) of the Schwarzschild-de Sitter metric and Friedmann Equations due to these **qgsc**. Indeed,substituting the expression $a(\eta)_q$ instead of a into the Friedmann Equation ((2.4) in [28]) without term with curvature

$$\frac{a'^2}{a^4} = \frac{8\pi}{3} G\rho, \quad (51)$$

we can obtain the Quantum Deformation (**QD**) [21] of the Friedmann Equation due to **qgcs** for **pbh** in the early Universe

$$\frac{a_q'^2}{a_q^4} = \frac{a'^2}{\exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) a^4} = \frac{8\pi}{3} G\rho \quad (52)$$

or

$$\frac{a'^2}{a^4} = \frac{8\pi}{3} G\rho \exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) \doteq \frac{8\pi}{3} G\rho_q,$$

$$\rho_q \doteq \rho \exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) < \rho. \quad (53)$$

The last line in (53) is associated with the fact that the Lambert W-function $W(u)$ is negative for $u < 0$.
Similarly, (ij)-components of the Einstein equations ((2.5) in [28])

$$2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\frac{8\pi}{3}Gp \tag{54}$$

within the foregoing (**QD**) are replaced by

$$2\frac{a''_q}{a^3_q} - \frac{a'^2_q}{a^4_q} = -\frac{8\pi}{3}Gp \tag{55}$$

or

$$\begin{aligned} 2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\frac{8\pi}{3}Gp\exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) = -\frac{8\pi}{3}Gp_q, \\ p_q \doteq p\exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) < p. \end{aligned} \tag{56}$$

It should be noted that the equation of the covariant energy conservation for the homogeneous background ((2.6) in [28])

$$\rho' = -3\frac{a'}{a}(\rho + p) \tag{57}$$

remains unaltered with replacement of $\rho \mapsto \rho_q, p \mapsto p_q$.
So, in the pattern of **3.1** (*the stationary pattern*), taking into consideration of **qgcs** for **pbhs** in the pre-inflationary era increases the initial values of the density ρ and of the pressure p in Friedmann equations.
The above calculations are correct if, from the start, we assume that a black hole (i.e., its event-horizon radius) is invariable until the onset of inflation. But such a situation is idealized because this period is usually associated with the radiation and absorption processes
Then again for $\mu = const$ from formulae (19),(18) we have

$$\begin{aligned} H_{0,q} = H_0\frac{r_{M_{orig}}}{r_{M_{orig,q}}}, \\ a(\eta)_q = a(\eta)\frac{r_{M_{orig,q}}}{r_{M_{orig}}}. \end{aligned} \tag{58}$$

Substituting the expression $a(\eta)_q$ from formula (58) in all formulae (52)–(57) we obtain analogues of these formulae in the general case. In particular, for formula (52) we have

$$\frac{a_q'^2}{a_q^4} = \frac{r_{M_{orig}}^2}{r_{M_{orig,q}}^2} \frac{a'^2}{a^4} = \frac{8\pi}{3} G\rho \tag{59}$$

Or, equivalently,

$$\frac{a'^2}{a^4} = \frac{8\pi}{3} G\rho \frac{r_{M_{orig,q}}^2}{r_{M_{orig}}^2} = \frac{8\pi}{3} G\rho_q$$
$$\rho_q \doteq \frac{r_{M_{orig,q}}^2}{r_{M_{orig}}^2} \rho. \tag{60}$$

In the same way as for formula (55), in this pattern for the general quantum deformation (ij)-components of Einstein equations by substitution of the value for $a(\eta)_q$ from the formula (58) we obtain

$$2\frac{a_q''}{a_q^3} - \frac{a_q'^2}{a_q^4} = -\frac{8\pi}{3} Gp \tag{61}$$

or

$$2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\frac{8\pi}{3} Gp \frac{r_{M_{orig,q}}^2}{r_{M_{orig}}^2} = -\frac{8\pi}{3} Gp_q,$$
$$p_q \doteq \frac{r_{M_{orig,q}}^2}{r_{M_{orig}}^2} p. \tag{62}$$

It is clear that, in this most general pattern, the covariant energy conservation for the homogeneous background ((2.6) in [28])

$$\rho' = -3\frac{a'}{a}(\rho + p) \tag{63}$$

remains unaltered with replacement of $\rho \mapsto \rho_q, p \mapsto p_q$.

6 The Quantum Fluctuations and Cosmological Perturbations Corrections Generated by qgcs for qpbhs. Non-Gaussianity Enhancement. The Onset

6.1 General Remarks

It is known that inflationary cosmology is characterized by *cosmological perturbations* of different nature (scalar, vector, tensor) [28],[30],[31], though vector perturbations are usually ignored as they die out fast.
It is clear that, as **qgcs** for **pbhs** in the early Universe cause shifts of the inflationary parameters, they inevitably lead to corrections of the cosmological perturbations on inflation. (14)
The following Remark, with the inferences used in the previous section, holds true:

Remark 6.1
From the above formulae it immediately follows that these shifts are arising only in the monomials, where the total power of the scale factor a (or Hubble parameter H) and of any derivatives with respect to η is not equal to 0. In this case the corresponding shifts are calculated using formulae (21)–(26) and their analogues.

6.2 Commentary on Corrections for Quantum Fluctuations and Cosmological Perturbations

Specifically, in the case of scalar cosmological perturbations consideration of the indicated **qgcs** for the rest of the Einstein equations (formulae (2.74)–(2.76) in [28]) in case **3.1** (*the stationary picture*) in virtue of **Remark 6.1** gives

$$\Delta\Phi - 3\frac{a'}{a}\Phi' - 3\frac{a'^2}{a^2}\Phi = 4\pi G a^2 \exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\cdot\delta\rho_{tot};$$
$$\Phi' + \frac{a'}{a}\Phi = -4\pi G a^2 \exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\cdot[(\rho+p)\upsilon]_{tot};$$
$$\Phi'' + 3\frac{a'}{a}\Phi' + (2\frac{a''}{a} - \frac{a'^2}{a^2})\Phi = 4\pi G a^2 \exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\cdot\delta p_{tot}. \qquad (64)$$

Here in the right sides of all lines in the last formula the scale factor a is taken with regard to **qgcs** from formula (22), i.e., $a = a(\eta)_q$. In the left sides of these lines additional factors of the type $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right), \exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right), \ldots$ are cancelled out because they are independent of η. This is so in the general case when taking in consideration **qgcs** for the **pbhs** formed in the pre-inflationary era (for all types of the cosmological perturbations, not only for those of the scalar type).
According to this remark, under the linearized form of the gauge transformations (formulae (2.31) in [28]), spatial components of the metric perturbation transform are retained due to inclusion of **qgcs** ([28],p.30):

$$\widetilde{h}_{ij} = h_{ij} - 2\partial_i\partial_j\sigma - \frac{a'}{a}\delta_{ij}\sigma'. \qquad (65)$$

And **qgcs** deform correspondingly the metric with scalar perturbations in the conformal Newtonian gauge (formulae (2.69) in [28]):

$$\{ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2]\} \mapsto a^2(\eta)_q[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2] =$$
$$= a^2(\eta)\exp\left(W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\left[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2\right].(66)$$

At the same time, the well-known Mukhanov–Sasaki equation [32],[33],[34]

$$\frac{1}{z}\frac{d^2z}{d\eta^2} = 2a^2H^2(1+\epsilon - \frac{3}{2}\widetilde{\eta} + \frac{1}{2}\widetilde{\eta}^2 - \frac{1}{2}\epsilon\widetilde{\eta} + \frac{1}{2H}\frac{d\epsilon}{dt} - \frac{1}{2H}\frac{d\widetilde{\eta}}{dt}), \qquad (67)$$

holds for these **qgcs**.
This is inferred from the fact that formula (26) may be completed with two

lines

$$\dot{\epsilon} \sim H;$$
$$\dot{\tilde{\eta}} \sim H. \tag{68}$$

As usual, in (67) $z \equiv a\dot{\phi}/H$ [34].
Then, for convenience to emphasize the use of quantum **pbh** with the mass M from the start, we introduce the following designations:

$$a(\eta)_M \doteq a(\eta)_q, H_M \doteq H_q; V_M \doteq V_q, ... \tag{69}$$

Let us consider, as in [35], quantum fluctuations of a generic scalar field during the de Sitter stage (p.121 in [35]). Similar to section 2.3 of [35], we represent the scalar field ϕ_0 in the form $\chi(\tau, \mathbf{x})$ as follows:

$$\chi(\tau, \mathbf{x}) = \chi(\tau) + \delta\chi(\tau, \mathbf{x})\,, \tag{70}$$

where $\chi(\tau)$ is the homogeneous classical value of the scalar field, $\delta\chi$ are its fluctuations and, in line with [35], τ is the the conformal time instead of η. Then, specifically for quantum fluctuations of a generic scalar field during the de Sitter stage (section 2.3 in [35]), the Klein–Gordon equation, which gives in an unperturbed FRW Universe (formula (40) in [35])

$$\chi'' + 2\mathcal{H}\chi' = -a^2\frac{\partial V}{\partial \chi}, \tag{71}$$

due to **Remark 6.1**, remains unaltered with deformation (21)–(26),(69)

$$\{\chi'' + 2\mathcal{H}_M\chi' = -a_M^2\frac{\partial V_M}{\partial \chi}\} \equiv \{\chi'' + 2\mathcal{H}\chi' = -a^2\frac{\partial V}{\partial \chi}\}. \tag{72}$$

Here in the usual way we have $\mathcal{H} = a'/a = a'_M/a_M = \mathcal{H}_M$.
But the above-mentioned deformation generated by **qgcs** for quantum **pbhs** not always retains the physical quantities arising in this consideration.
Because in this case such an important quantity as the power-spectrum acquires multiplicative increments. In particular, in the case of the de Sitter

phase and of a very light scalar field χ, with $m_\chi \ll 3/2H$ (formula (64) in [35]), the power-spectrum on superhorizon scales in this pattern is changed

$$\{\mathcal{P}_{\delta\chi}(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{3-2\nu_\chi}\} \Rightarrow \{\mathcal{P}_{\delta\chi,M}(k) = \left(\frac{H_M}{2\pi}\right)^2 \left(\frac{k}{a_M H_M}\right)^{3-2\nu_\chi}\} = \\ = \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\mathcal{P}_{\delta\chi}(k). \quad (73)$$

In the last formula we use the obvious equality to the condition $aH = a_M H_M$ and the fact that the quantity ν_χ satisfies the condition (formula (57) in [35])

$$\frac{3}{2} - \nu_\chi \simeq \eta_\chi = (m_\chi^2/3H^2). \quad (74)$$

As for the effective mass m_χ of the scalar field χ we have the equality $m_\chi^2 = \partial^2 V/\chi^2$, from formula (25) it follows that the exponent $3 - 2\nu_\chi$ in formula (73) is also retained with this deformation. Since the quantity $\exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > 1$, with due regard for **qgcs** in the de Sitter stage, magnitude of the power-spectrum is growing.
Similar results are true also for the case of quantum fluctuations of a generic scalar field in the quasi–de Sitter stage (Section 2.4 in [35]). We obtain an analog of the shift in formula (73)

$$\{\mathcal{P}_{\delta\chi}(k) \simeq \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{3-2\nu_\chi}\} \Rightarrow \{\mathcal{P}_{\delta\chi,M}(k) \simeq \left(\frac{H_M}{2\pi}\right)^2 \left(\frac{k}{a_M H_M}\right)^{3-2\nu_\chi} \simeq \\ \simeq \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\mathcal{P}_{\delta\chi}(k)\}. \quad (75)$$

Though, as distinct from 74), formula (72) in [35] is true

$$\frac{3}{2} - \nu_\chi \simeq \eta_\chi - \epsilon, \quad (76)$$

where ϵ is the deceleration parameter from formula (27) characterized by $\epsilon = \epsilon_M \doteq \epsilon_q$.From this it follows that formula (75) is valid, whereas the power-spectrum for quantum fluctuations of a generic scalar field in the quasi–de Sitter stage is growing similar to the de Sitter stage.

Remark 6.2
From **Remark 2.3** it directly follows that right sides of formulae (73),(75)are series expanded in terms of the small parameter α_{r_M}:

$$\mathcal{P}_{\delta\chi,M}(k) \simeq \exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)\mathcal{P}_{\delta\chi}(k) = \\ = (1+\omega_{\mathbf{1}}\alpha_{r_M}+\omega_{\mathbf{2}}\alpha_{r_M}^2+\omega_{\mathbf{3}}\alpha_{r_M}^3+...)P_{\delta\chi}(k), \quad (77)$$

where the first (leading) term in this series corresponds to the initial value of this power-spectrum (without the indicated **qgcs**), whereas the other terms – to the corrections of different orders for these **qgcs**, which are the greater the lower the size of the initial **pbh**, i.e. the lower the mass M (or same the radius r_M).
From formula (9) we can easily derive explicit values of the coefficients $\omega_i, i \geq 1$. Indeed, because

$$(1+\omega_{\mathbf{1}}\alpha_{r_M}+\omega_{\mathbf{2}}\alpha_{r_M}^2+\omega_{\mathbf{3}}\alpha_{r_M}^3+...) = \left(1+\frac{1}{2e}\alpha_{r_M}+\frac{5}{8e^2}\alpha_{r_M}^2+\frac{49}{48e^3}\alpha_{r_M}^3+\ldots\right)^2, (78)$$

we have $\omega_{\mathbf{1}} = 1/e, \omega_{\mathbf{2}} = 3/2e^2, ...$
In the right side of (78) we have the squared series from the right side of formula (9) in terms of the small parameter α_{r_M}.
It is clear that we can also express as a series in terms of a dimensionless small parameter α_{r_M} the right sides in other formulae too, specifically in (52)–(56),(64),(66), etc. In every case, a part of this series going after the leading term detects the deviation from the semi-classical approximation mode in the approach to quantum gravity within the scope of GUP (6).

6.3 Enhancement of Non-Gaussianity

It is understandable, when non-Gaussianities are involved in the process of inflation, the above-mentioned **qgcs** for quantum **pbhs** should change the parameters of these non-Gaussianities. It is required to know what are the changes. Deviation from the Gaussian distribution for the random field $g(\mathbf{x})$ is a nonzero value of the three-point correlator ((6.30) in [34])

$$\langle g_{\mathbf{k_1}} g_{\mathbf{k_2}} g_{\mathbf{k_3}} \rangle = (2\pi)^3 \delta^3_{\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}} B_g(k_1, k_2, k_3), \tag{79}$$

where $g_{\mathbf{k_i}}, i = 1, 2, 3$ represent the Fourier transform $g(\mathbf{x})$ in the momentum k_i and the quantity B_g, named the *bispectrum*, is a measure of non-Gaussinity for the random field $g(\mathbf{x})$.
We can show that, with due regard for the foregoing **qgcs**, the absolute value of the *bispectrum* B_g in the inflation pattern for different random fields is growing.
Let us consider several examples:

6.3.1 Non-Gaussianity qgcs-Correction of Field Perturbations in Different Patterns

Non-Gaussianity correction from the self-interaction of the field

In this case the non-Gaussianity arises from the self-interaction of the field ([36] and **24.4.2** in [34]). Neglecting the metric perturbation at the initial stage and considering the effect of keeping the quadratic term in the perturbed field equation, we can obtain (formula (24.39) in [34]) the following:

$$\ddot{\delta\phi} + 3H_* \dot{\delta\phi} - a^{-2}\nabla^2 \delta\phi + \frac{1}{2} V_*^{'''} (\delta\phi)^2 = 0. \tag{80}$$

Here the asterisk denotes a value of the corresponding quantity during inflation which is taken to be constant, and the dot, as usual, denotes $\partial/\partial t$. Then the quantity B_g in this case (formula (24.43) in [34]) takes the form

$$B^{self}_{\delta\phi} \sim H_*^2 V_*^{'''}. \tag{81}$$

However, considering the **qgcs** under study, due to formulae (24),(26) and (11),(12), the quantities in the right side of the last formula are transformed as follows:

$$H_*^2 \rightarrow H_{*,M}^2 = \exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) H_*^2;$$
$$V_*^{'''} \rightarrow V_{*,M}^{'''} = \exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)) V_*^{'''}. \tag{82}$$

So, considering these **qgcs**, we find that the *bispectrum* $B^{self}_{\delta\phi,M}$ in this case is

$$B^{self}_{\delta\phi,M} = \exp\left(-2W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) B^{self}_{\delta\phi}. \quad (83)$$

As $\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) > 1$,from the last relation it follows that

$$|B^{self}_{\delta\phi,M}| > |B^{self}_{\delta\phi}|. \quad (84)$$

This means that with regard to **qgcs** for quantum **pbhs** non-Gaussianity is enhanced.
According to **Remark 6.2**, $\exp\left(-2W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)$ can also be series expanded in terms of the dimensionless small parameter α_{r_M} by squaring of the left side in the formula (78) and by calculating the coefficients for the corresponding powers α_{r_M}. As noted in **Remark 6.2**, nontrivial terms of this series generate **qgcs** to the semiclassical *bispectrum* $B^{self}_{\delta\phi}$.

The Non-Gaussianity Correction of Field Perturbations from Gravitational interactions

In the above analysis of non-Gaussianity there was no metric perturbation because the gravitational interaction was"excluded". When it is taken into consideration, the non-Gaussianity in this case arises from the metric perturbation and the *bispectrum* takes the following form ([37] and formula (24.46) in [34]):

$$B^{grav}_{\delta\phi} \sim -\frac{1}{8}H^4_*(\frac{V'}{V}). \quad (85)$$

With regard to **qgcs**, from formula (24) we can derive

$$B^{grav}_{\delta\phi} \rightarrow B^{grav}_{\delta\phi,M} = \exp\left(-2W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) B^{grav}_{\delta\phi}. \quad (86)$$

Note that (86) is valid only in the case when **qgcs**-correction of the metric is not taken into consideration or such correction is considered to be vanishingly small. In this case from formulae (83),(86) we can obtain

$$\frac{B^{self}_{\delta\phi,M}}{B^{self}_{\delta\phi}} = \frac{B^{grav}_{\delta\phi,M}}{B^{grav}_{\delta\phi}} = \exp\left(-2W\left(-\frac{1}{e}\alpha_{r_M}\right)\right). \tag{87}$$

Based on the last formula, it can be concluded that
growth of non-Gaussianity of the field perturbations in inflation generated by **qgcs** *for a primordial black hole in the pre-inflation era is independent of the pattern (with or without regard for the metric perturbation) considered if* **qgcs***-correction of the metric may be neglected.*

6.3.2 Non-Gaussianity qgcs-Correction for the Tensor Primordial Perturbations

As seen, the right side of (87) arises with regard to the **qgcs**-correction as a factor of the enhanced non-Gaussianity and the tensor primordial perturbations.
Actually, denoting in this case *bispectrum* as $B_{h^{TT}_{ij}}$, where by formula (5.43) in [28] we have

$$h^{TT}_{ij}(\eta, \mathbf{k}) = \sum_{A=+,\times} e^{(A)}_{ij} h^{(A)}(\eta, \mathbf{k}), \tag{88}$$

in the left side the symmetric transverse traceless tensor h^{TT}_{ij} ((2.42) in [28]) has helicity 2, whereas the right side represents expansion of this tensor by the sum over polarization.
According to formulae (1.1) in [38] and (24.63) in [34], we have

$$B_{h^{TT}_{ij}} \sim \frac{H^4_*}{m^4_p}. \tag{89}$$

Whence, in analogy with formula (87), it directly follows that

$$\frac{B_{h^{TT}_{ij},M}}{B_{h^{TT}_{ij}}} = \exp\left(-2W\left(-\frac{1}{e}\alpha_{r_M}\right)\right). \tag{90}$$

From the start In this section it has been assumed that this quantum **pbh** in the pre-inflationary era is considered in the state **3.1**. *the stationary* *pattern.* Still, we can obtain similar results for other processes proceeding before the onset of inflation, specifically for **3.3** *black hole evaporation.*

7 Conclusion and Further Steps

In this way it has been demonstrated that, within the scope of natural assumptions, the **qgcs** calculated for **pbhs** arising in the pre-inflationary epoch contribute significantly to the inflation parameters, enhancing non-Gaussianity in the case of cosmological perturbations. Besides, withy due regard for these **qgcs**, the probability of arising **pbhs** is higher.
Based on the results of this paper, the following steps may be planned to study the corrections of cosmological parameters and cosmological perturbations due to **qgcs** for **pbhs** in the pre-inflationary era:

7.1 Comparison of the results obtained in Section 6 with the experimental data accumulated by space observatories: (Planck Collaboration), (WMAP Collaboration) [39],[40],[41].

7.2 Elucidation of the fact, how closely the author's results are related to general approaches to inclusion of the quantum-gravitational effects in studies of inflationary perturbations (for example, [42]–[45]);

7.3 Elucidation of the possibility to extend the obtained results to other types of quantum **pbhs**, in particular to **pbh** with the mass M, with the electric charge Q but without rotation in the Reissner-Nordström (RN) Metric (for the normalization $c = \hbar = G = 1$) [13],[8]

$$ds^2 = \left(1 + \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 d^2\Omega. \tag{91}$$

IN this case the event horizon radius $r_{\pm,RN}$ and the temperature T_{RN} of such a hole respectively take the following forms: [13],[8]

$$r_{\pm,RN} = M \pm \sqrt{M^2 - Q^2}. \tag{92}$$

and

$$T_{RN} = \frac{(M^2 - Q^2)^{1/2}}{2\pi \left(M + (M^2 - Q^2)^{1/2} \right)^2}. \tag{93}$$

Selecting in formula (92) for the radius r_{RN} of such **pbh** the value $r_{RN} = r_+$ and assuming, similar to [46], that $Q/M \ll 1$, we can obtain that the (RN) metric (91) for small M represents, to a high accuracy, the Schwarzschild metric (2) and the Schwarzschild-de Sitter metric (3) with the corresponding formulae for the event horizon radius r_{RN} and for the temperature T_{RN}, which are close to the corresponding Schwarzschid's from Section 2. In this way for the quantum PBH with (RN) metric derivation of the results similar to those given in this work (inclusion of **qgcs**) is relevant, at least for the case $Q/M \ll 1$.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this work.

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