Is neutrino produced in standard weak interactions a Dirac or Majorana particle?

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Abstract

This work considers the following problem: what type (Dirac or Majorana) of neutrinos is produced in standard weak interactions? It is concluded that only Dirac neutrinos but not Majorana neutrinos can be produced in these interactions. Then neutrino interacts with $W^{\pm}$ and Z bosons but neutrinoless double beta decay is absent. It means that this neutrino will be produced in another type of interaction. Namely, Majorana neutrino will be produced in the interaction which differentiates spin projections but cannot differentiate neutrino (particle) from antineutrino (antiparticle). Then neutrino will interact with $W^{\pm}$ bosons and neutrinoless double beta decay will arise. But standard interaction with Z boson will be absent. Such interaction has not been discovered yet. Therefore experiments with very high precision are important to detect the neutrinoless double decay if they are realized in the Nature.

1   Introduction

The standard model [1] is composed by using Dirac fermions [2]. At present time violation of the electron (l_{ν_e}), muon (l_{ν_μ}), tau (l_{ν_τ}) numbers have been detected, then only the common lepton number is conserved but not every lepton number individually. Then there is a question: Does full violation of lepton numbers take place? In this case the Majorana neutrino [3] can be realized where the conservation lepton number does not appear. Realization of the Majorana neutrino can be fulfilled in two ways:

1. To suppose that a priori neutrino is a Majorana particle, i. e., to work in the framework of the standard model where it is presumed that neutrinos are Majorana particles.

2. To search for a condition (interaction) when Majorana neutrino is produced.

Almost in all works [4] ÷ [9] where the Majorana neutrino is investigated the first approach is used and there is a conclusion that there is no possibility to differ is neutrino a Majorana or Dirac particle, except the case of neutrinoless double beta decay. In work [10] it has been shown that for chiral invariance of the standard weak interactions the neutrinoless double beta decay cannot be realized.

In this work we will use the second approach. In means that it is necessary to find when and how (or in which interactions) the Majorana neutrino can be produced. This approach is based on the fundamental physical principle that in every interaction the particles are produced in eigenstates. Then our aim is to find in which type of interactions the Majorana neutrino can be produced as eigenstate.

Now come to a consideration of properties of Dirac and Majorana particles.

2   Dirac neutrino

The equation for a particle with spin $\frac{1}{2}$ was first formulated by Dirac [2] in 1928. Afterwards it turned out that this representation was adequate to describe neutral and charged fermions, i. e., fermions are Dirac particles. Dirac equation for free fermion (spinor with spin $\frac{1}{2}$) has the following form:

$$(\gamma^\mu p_\mu - m)\psi = 0, \tag{1}$$

where γ^μ- are Dirac matrices, p_μ -are energy and impulse of the particle, m - is a particle mass and ψ is the fermion wave function.

If to introduce the following projecting operators

$$P_L = \frac{(1-\gamma^5)}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad P_R = \frac{(1+\gamma^5)}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2}$$

which form the full system

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = 0, \tag{3}$$

then we can write ψ in the form

$$\psi = \psi_R + \psi_L, \tag{4}$$

where

$$\psi_R = P_R\psi, \quad \psi_L = P_L\psi,$$

are the left and right spin projections.

Above we have considered free fermions, then to consider their productions, annihilations and et cetera, it is necessary to introduce their interactions. For introducing interactions in the framework of the quantum field theory [11] we use derivative lengthening and the principle of gauge invariance. For example, in the electrodynamics:

$$p_\mu \to p_\mu - ieA_\mu(x), \quad A'_\mu(x) = A_\mu(x) + \alpha(x), \tag{5}$$

$$\bar{\psi}' = \bar{\psi}\exp(i\alpha(x)), \quad \psi' = \psi\exp(i\alpha(x)), \tag{6}$$

where A_μ is a vector field, e- a couple constant of electromagnetic interactions, α - a phase of gauge transformation. Then equation (1) is invariant relative to the above gauge transformation. The Lagrangian of electromagnetic interactions is

$$L(\bar{\psi}, \psi, A_\mu) = -ie\bar{\psi}\gamma^\mu\psi A_\mu \equiv -ie(\bar{\psi}_L + \bar{\psi}_R)\gamma^\mu(\psi_L + \psi_R)A_\mu. \tag{7}$$

In analogous manner we also introduce interactions in the strong interaction [12], [13] and the electroweak model [1]. In electromagnetic and strong interactions the left and right components of fermions participate in a symmetric manner and

$$\psi = \psi_L + \psi_R,$$

while left doublets and right singlets of fermions participate in weak interactions [1] (in charged current on weak interactions only left components of fermions participate). The Lagrangian of the interaction of the electron neutrino with the electron-positron (charged leptons) by W bosons, has the following form:

$$L(W) = -\frac{ig_W}{\sqrt{2}}[\bar{\nu}_{Le}\gamma^\mu e_L W_\mu^+ + \bar{e}_L\gamma^\mu\nu_{Le}W_\mu^-]. \tag{8}$$

For neutrinos it means that only the left components of neutrinos and antineutrinos participate in the weak interactions.

Experiments [14] were performed to measure the neutrino spirality. The result is the following - neutrino has the left spirality and antineutrino has the right spirality. These characteristic of neutrino was used to formulate the electroweak model [1].

So, in electromagnetic and strong interactions the left and right components of fermions (quarks and charged leptons) participate in a symmetric form:

$$\bar{\psi} = \bar{\psi}_L + \bar{\psi}_R, \quad \psi = \psi_L + \psi_R, \tag{9}$$

while only the left components of quarks and leptons participate in the weak interactions:

$$\bar{\psi} \to \bar{\psi}_L + 0, \quad \psi \to \psi_L + 0,$$

i. e. for neutrino

$$\bar{\nu} \equiv \bar{\nu}_L, \quad \nu \equiv \nu_L. \tag{10}$$

So, if neutrino is Dirac particle then there are interactions (by using the principle of gauge invariance) with $W^\pm$ and Z bosons but neutrinoless double beta decay is absent.

3   Majorana neutrino

In 1937 Majorana found an equation [3] for a fermion with spin $\frac{1}{2}$. Then it became clear that this fermion could be only a neutral particle since the particle and antiparticle are joined in one representation. The Majorana equation for neutrino is [3]

$$i(\hat{\sigma}^\mu d_\mu)\nu_R - m_R^M \epsilon \nu_R^* = 0,$$
$$i(\hat{\sigma}^\mu d_\mu)\nu_L - m_L^M \epsilon \nu_L^* = 0, \tag{11}$$

where $\hat{\sigma}^\mu \equiv (\sigma^0, \sigma)$, $\sigma^\mu \equiv (\sigma^0, -\sigma)$, σ is Pauli matrices,

$$\epsilon = \begin{pmatrix} 0, 1 \\ -1, 0 \end{pmatrix}.$$

These equations describe two completely different neutrinos with masses m_R^M and m_L^M which do not possess any additive numbers and neutrinos are their own antineutrinos; i. e., particles differ from antiparticles only in spin projections. Now it is possible to introduce the following two Majorana neutrino states:

$$\chi_L = \nu_L + (\nu_L)^c,$$
$$\chi_R = \nu_R + (\nu_R)^c. \tag{12}$$

$$(\nu_{LR})^c = C\bar{\nu}_{LR}^T,$$

where C - is a matrix of charge conjugation, T - is transposition [4]-[6]. Formally the above Majorana equation (11) can be rewritten in the following form:

$$(\gamma^\mu \partial_\mu + m)\chi(x) = 0, \tag{13}$$

with the Majorana condition ($\chi \equiv \chi_{LR}$)

$$C\bar{\chi}^T(x) = \xi\chi(x), \tag{14}$$

where ξ is a phase factor ($\xi = \pm 1$)

It is necessary to stress that

$$\bar{\chi}(x)\gamma^\mu \chi(x) = 0; \tag{15}$$

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i. e., vector current of Majorana neutrino is equal to zero.

In 1950s the Majorana neutrino study was very extensive [15]. Later it stopped. By that time the spirality of neutrinos had been measured [14].

From the above consideration now we know that in strong and electromagnetic interactions the left and right components of fermions participate symmetrically while in the weak interactions only the left components of fermions participate (in neural current the right components of fermions are also present, but in a nonsymmetric form).

What is a Majorana neutrino? As it has been stressed above, that Majorana particle χ with spin $\frac{1}{2}$ has two spin $\pm\frac{1}{2}$ projections. The (left) component with projection $-\frac{1}{2}$ is correlated with neutrino ν_L and the (right) component with projection $+\frac{1}{2}$ is correlated with antineutrino $(\nu_L)^c$. Here is an analogy with the electromagnetic or strong interactions (see exp. (7)), where the left and right components of fermions participate symmetrically. In order to produce the Majorana neutrino, we must have an interaction where neutrino and antineutrino participate symmetrically. From the experimental data we have known that in the weak interactions neutrino and antineutrino are produced in separate processes but not in one process symmetrically. So, Majorana neutrino can be produced only in the interaction which does not differentiate particle from antiparticle but differentiates spin projections of the particles (fermions). By analogy with electromagnetic interactions we can introduce Majorana neutrino current in the following form:

$$j^\mu = \bar{\chi}(x)\gamma^\mu\chi(x), \tag{16}$$

but as it is stressed in exp. (15) this value is zero. We can introduce the gauge transformation for the following Majorana neutrinos χ_L and χ_R:

$$\begin{aligned} \chi'_L &= \exp(-i\beta)\chi_L \\ \chi'_R &= \exp(i\beta)\chi_R \end{aligned}, \tag{17}$$

then we can also introduce Majorana charge g_M. It is clear that this charge will differ from Dirac charge g_W. If this Majorana particle interacts with a Dirac particle it must have the Dirac charge (i.e., Majorana neutrino must have a double charge). Since usually it is supposed that masses of χ_L and χ_R neutrino differ very much then this gauge transformation must be strongly violated.

A reaction with the double beta decay with two electrons

$$(Z, A) \rightarrow (Z + 2, A) + e_1^- + e_2^- + \bar{\nu}_{e1} + \bar{\nu}_{e2}, \tag{18}$$

is possible if $M_A(Z, A) > M_A(Z + 2, A)$.

If neutrino is a Majorana particle $(\chi_L = \nu_L + (\nu_L)^c)$, then the following neutrinoless double beta decay is possible:

$$(Z, A) \rightarrow (Z + 2, A) + e_1^- + e_2^-, \tag{19}$$

if $M_A(Z, A) > M_A(Z + 2, A)$.

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The lepton part of the amplitude of the above two neutrino decay has the following form [5], [6], [8] :

$$\bar{e}(x)\gamma_\rho\frac{1}{2}(1\pm\gamma_5)\nu_j\bar{e}(y)\gamma_\sigma\frac{1}{2}(1\pm\gamma_5)\nu_k(y). \tag{20}$$

After substituting the Majorana neutrino propagator and its integrating on the momentum of virtual neutrino, the lepton amplitude gets the following form:

$$-i\delta_{jk}\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq(x-y)}}{q^2-m_j^2}\bar{e}(x)\gamma_\rho\frac{1}{2}(1\pm\gamma_5)(q^\mu\gamma_\mu+m_j)\frac{1}{2}(1\pm\gamma_5)\gamma_\sigma e(y). \tag{21}$$

If we use the following expressions:

$$\frac{1}{2}(1-\gamma_5)(q^\mu\gamma_\mu+m_j)\frac{1}{2}(1-\gamma_5)=m_j\frac{1}{2}(1-\gamma_5), \tag{22}$$

$$\frac{1}{2}(1-\gamma_5)(q^\mu\gamma_\mu+m_j)\frac{1}{2}(1+\gamma_5)=q^\mu\gamma_\mu\frac{1}{2}(1+\gamma_5), \tag{23}$$

then we see that in the case when there are only left currents (expression (22)) we get a deposit only from the neutrino mass part,

$$-i\delta_{jk}\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq(x-y)}}{q^2-m_j^2}\bar{e}(x)\gamma_\rho\frac{1}{2}(1-\gamma_5)(q^\mu\gamma_\mu+m_j)\frac{1}{2}(1-\gamma_5)\gamma_\sigma e(y)=$$

$$=-i\delta_{jk}\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq(x-y)}}{q^2-m_j^2}\bar{e}(x)\gamma_\rho m_j\frac{1}{2}(1-\gamma_5)\gamma_\sigma e(y), \tag{24}$$

while at the presence of the right currents (expression (23))

$$-i\delta_{jk}\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq(x-y)}}{q^2-m_j^2}\bar{e}(x)\gamma_\rho\frac{1}{2}(1-\gamma_5)(q^\mu\gamma_\mu+m_j)\frac{1}{2}(1+\gamma_5)\gamma_\sigma e(y)=$$

$$=-i\delta_{jk}\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq(x-y)}}{q^2-m_j^2}\bar{e}(x)\gamma_\rho q^\mu\gamma_\mu\frac{1}{2}(1+\gamma_5)\gamma_\sigma e(y), \tag{25}$$

the amplitude includes the term proportional to four momentum q in the neutrino propagator.

So, we see that if neutrino is a Majorana particle, then the neutrinoless double decay will take place. As we have shown above, the standard weak interactions can produce only Dirac neutrinos. The Majorana neutrino can produce only the interaction which does not differentiate neutrino from antineutrino and, then neutrino and antineutrino are left and right spin projections of one particle. Until now this interaction has not been discovered. Detection of the neutrinoless double decay will be an indication on the existence of such interaction, therefore experiments with very high precision are needed. Consideration of the problem, in what concrete interaction the Majorana neutrino can be produced, will be continued in subsequent works.

So, if neutrino is Majorana particle then there are interactions with $W^{\pm}$ bosons (constructed by hand). The standard interaction with Z boson is absent (see expression (15)) but then neutrinoless double beta decay arises. Obviously from the available experimental data we can come to conclusion that neutrino produced in the standard weak interactions are Dirac particles.

4   Conclusion

In this work the next problem was considered: what type (Dirac or Majorana) of neutrinos is produced in the standard weak interactions? We have come to a conclusion that these interactions can produce only Dirac neutrinos but not Majorana's. Then neutrino interacts with $W^{\pm}$ and Z bosons but neutrinoless double beta decay is absent. It means that this neutrino will be produced in another type of interaction. Namely, Majorana neutrinos will be produced in the interaction which differentiates spin projections but cannot distinguish neutrino (particle) from antineutrino (antiparticle). Then neutrino will interact with $W^{\pm}$ bosons and neutrinoless double beta decay will arise. But the standard interaction with Z boson will be absent. Such interactions have not been discovered yet. Therefore experiments with very high precision are needed to detect the neutrinoless double decay if they are realized in the Nature.

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