The Gravity on All Energy Scales. Some Significant Examples and One No-Go Theorem

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PACS: 04.20.Cv, 04.60.Bc
Keywords:gravity,fundamental length,discrete parameters

Abstract

At the present time a theory of gravity is subdivided into two absolutely different parts: low-energy theory represented by the General Relativity (GR) and hypothetical high-energy theory – Quantum Gravity (QG) – that is still unresolved. In this way there is a certain dichotomy in gravity considered as a unified theory. This work is an effort to reveal the main causes for such a dichotomy; the means for departure from this dichotomy are proposed. By one of the approaches gravity is considered at low and at high energies as a single whole dependent on the same parameters, which are discrete for the fundamental length if present.There are grounds to believe that in this case the mathematical formalism must be modified – all infinitesimal space-time quantities must be replaced by the corresponding finite quantities dependent on the existent energies. Further this paper shows that, provided a theory involves the minimal length, the parameters associated with it will appear in several models of general relativity and cosmology. But smallness of these parameters and smoothness of their variation at low energies makes it possible to consider them practically continuous, the models themselves being in essence independent of the parameter variations. At high energies these parameters are really discrete and lead to equations with a discrete set of solutions. Consideration is given to some consequences

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and, in particular, to some differences between the, so far, hypothetical theory involving the minimal length and general relativity. Finally, one fairly evident no-go theorem is treated to demonstrate that the entropic approach to gravity in its present form is impossible in the case of the minimal length theory.

1 Introduction. Infinitesimal Quantities in Quantum Theory and Gravity, Their Measurability

At the present time, the mathematical apparatus of both special and general relativity theories (and of a quantum theory as well) is based on the concept of continuity and on analysis of infinitesimal spatial-temporal quantities. This is a corner stone for the Minkowski space geometry (MS) and also for the Riemannian geometry (RG) [1].
However, this approach involves a problem when we proceed to a quantum description of nature. Even at a level of the heuristic understanding, it is clear that, as measuring procedures in a quantum theory are fundamental, the description with the use of infinitesimal quantities is problematic because in its character the measuring procedure is discrete.
At a level of the mathematical formalism and physical principles, incompatibility of both the Minkowski space geometry and Riemannian geometry with the uncertainty principle is expected in any ≪format≫, in relativistic and nonrelativistic cases.
Indeed, in a nonrelativistic case, according to the Heisenberg's Uncertainty Principle (HUP) we have [2]:

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}. \tag{1}$$

Or for the pair energy - time we have

$$\Delta E \Delta t \geq \frac{\hbar}{2}, \tag{2}$$

And this is in conflict with the possibility of infinitesimal coordinate variations in RG (or General Relativity (GR)) because the mathematical formalism of GR is based on the concept of infinitesimal variations in the spatial-temporal quantities $ds, dx_\mu, ...$, which for a probe particle, in accordance with (1),(2), will inevitably result in the infinitely large momentum and energy fluctuations

$$\Delta p_i \rightarrow \infty; \Delta E \rightarrow \infty. \tag{3}$$

But then, if measurable quantities are concerned, (3) is in conflict with GR because, as immediately follows from (3), when measuring the characteristics for variations in the probe particle positions within the scope of GR and, actually, being at low energies, we can derive the momentum and energy characteristic for the scales of Quantum Gravity (QG)!
In a relativistic case for any probe particle with the mass m, if it is considered as a ≪**point object**≫, there is its Compton wavelength [3]

$$\bar{\lambda}_C = \frac{\lambda_C}{2\pi} = \frac{\hbar}{mc} \tag{4}$$

setting the ultimate accuracy for the determination of its coordinates. But, due to the infinitesimal special-temporal variations in MS, this minimum is easily gone beyond.
In any case, no matter relativistic or nonrelativistic, these infinitesimal variations of the space-time quantities $ds, dx_\mu, ...$ may be associated with ≪**nonmeasurable**≫, from the physical point of view, length intervals, that is the current mathematical formalism of MS and of RG (GR) in conjunction with the basic quantum principle leads to a **nonclosure** theory in conflict with both the physical principles and common sense.
Thus, the mathematical formalism utilizing the infinitesimal spatial-temporal quantities should be revised if we want to derive a theory **measurable** from the viewpoint of a Quantum Theory.
All the infinitesimal spatial-temporal quantities $ds, dx_\mu, ...$ should be replaced by the finite quantities dependent on the measuring scales l (energies $E \sim 1/l$):

$$ds^2 \mapsto \tau s^2(l); dx_\mu \mapsto \tau x_\mu(l), ... \tag{5}$$

On the other hand, it is well known that at high energies (on the order of the quantum gravity energies) the minimal length l_{min} to which the indicated energies are ≪sensitive≫, as distinct from the low ones, should inevitably become apparent in the theory. But if l_{min} is really present, it must be present at all the ≪Energy Levels≫ of the theory, low energies including. And this, in addition to the above arguments, points to the fact that the mathematical formalism of the theory should not involve any infinitesimal spatial-temporal quantities. Besides, some new parameters become involved, which are dependent on l_{min} [5]–[9].

What are the parameters of interest in the case under study? It is obvious that, as the quantum-gravitational effects will be revealed at very small (possibly Planck's) scales, these parameters should be dependent on some limiting values, e.g., $l_P \propto l_{min}$ and hence Planck's energy E_P.

This means that in a high-energy gravitation theory the energy- or, what is the same, measuring scales-dependent parameters should be necessarily introduced.

But, on the other hand, these parameters could hardly disappear totally at low energies, i.e. for GR too. However, since the well-known canonical (and in essence the classical) statement of GR has no such parameters [1], the inference is as follows: their influence at low energies is so small that it may be disregarded at the modern stage in evolution of the theory and of the experiment.

Still this does not imply that they should be ignored in future evolution of the theory, especially on going to its high-energy limit.

This paper presents a study of how the additional parameters associated with the minimal length l_{min} become naturally involved into some models of gravity and cosmology even at low energies and why at these energies their effect is so insignificant.

This work has some points of intersection in Sections 1 and 2 with [10] and [11]. The remaining sections present new results and inferences.

2 Quantum Fluctuations of Space-Time, Deformation and New Parameters

To solve the above-mentioned problems, **initially** we can use the Space-Time Quantum Fluctuations (STQF) imposing considerable constraints on

HUP with regard to gravity. The definition (STQF) is closely connected to the notion of ≪space-time foam≫.
The notion ≪space-time foam≫, introduced by J. A. Wheeler about 60 years ago for the description and investigation of physics at Planck's scales (Early Universe) [12],[13], is fairly settled. Despite the fact that in the last decade numerous works have been devoted to physics at Planck's scales within the scope of this notion, for example [15]–[34], by this time still their no clear understanding of the ≪space-time foam≫ as it is.
On the other hand, it is undoubtful that a quantum theory of the Early Universe should be a deformation of the well-known quantum theory.
In my works with the colleagues [35]–[43] I has put forward one of the possible approaches to resolution of a quantum theory at Planck's scales on the basis of the density matrix deformation.
In accordance with the modern concepts, the space-time foam [13] notion forms the basis for space-time at Planck's scales (Big Bang). This object is associated with the quantum fluctuations generated by uncertainties in measurements of the fundamental quantities, inducing uncertainties in any distance measurement. A precise description of the space-time foam is still lacking along with an adequate quantum gravity theory. But for the description of quantum fluctuations we have a number of interesting methods (for example, [14],[24]–[34]).
In what follows, we use the terms and symbols from [26]. Then for the fluctuations $\widetilde{\delta l}$ of the distance l we have the following estimate:

$$(\widetilde{\delta l})_{\gamma} \gtrsim l_P^{\gamma} l^{1-\gamma} = l_P (\frac{l}{l_P})^{1-\gamma} = l(\frac{l_P}{l})^{\gamma} = l\lambda_l^{\gamma}, \qquad (6)$$

or that same

$$|(\widetilde{\delta l})_{\gamma}|_{min} = \beta l_P^{\gamma} l^{1-\gamma} = \beta l_P (\frac{l}{l_P})^{1-\gamma} = \beta l \lambda_l^{\gamma}, \qquad (7)$$

where $0 < \gamma \leq 1$, coefficient β is of order 1 and $\lambda_l \equiv l_P/l$.
From (6),(7), we can derive the quantum fluctuations for all the primary characteristics, specifically for the time $(\widetilde{\delta t})_{\gamma}$, energy $(\widetilde{\delta E})_{\gamma}$, and metrics $(\widetilde{\delta g}_{\mu\nu})_{\gamma}$. In particular, for $(\widetilde{\delta g}_{\mu\nu})_{\gamma}$ we can use formula (10) in [26]

$$(\widetilde{\delta g}_{\mu\nu})_{\gamma} \gtrsim \lambda^{\gamma}. \qquad (8)$$

Further in the text is assumed that the theory involves a minimal length on the order of Planck's length

$$l_{min} \propto l_P$$

or that is the same

$$l_{min} = \xi l_P, \tag{9}$$

where the coefficient ξ is on the order of unity too.
In this case it is unimportant which is the origin of this minimal length. For simplicity, we assume that it comes from the Generalized Uncertainty Principle (GUP) that is an extension of HUP for Planck's energies, where gravity must be taken into consideration [44]–[51]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_P^2 \frac{\triangle p}{\hbar}. \tag{10}$$

Here α' is the model-dependent dimensionless numerical factor.
(10) leads to the minimal length $l_{min} = \xi l_P = 2\sqrt{\alpha'} l_P$.
Therefore, in this case replacement of Planck's length by the minimal length in all the above formulae is absolutely correct and is used without detriment to the generality

$$l_P \rightarrow l_{min}. \tag{11}$$

Thus, $\lambda_l \equiv l_{min}/l$ and then (6)– (8) upon the replacement (11) are read unchanged.
The following points of importance should be noted:
2.1)It is clear that **at Planck's scales, i.e. at the minimal length scales**

$$l \rightarrow l_{min} \tag{12}$$

models for different values of the parameter γ are coincident.

2.2)**Provided some quantity has a minimal measuring unit, values of this quantity are multiples of this unit**.
Naturally, any quantity having a minimal measuring unit is uniformly discrete.
The latter property is not met, in particular, by the energy E.

As $E \sim 1/l$, where l – measurable scale, **the energy E is a discrete quantity but the nonuniformly discrete one**. It is clear that the difference between the adjacent values of E is the less the lower E. In other words, for

$$E \ll E_P \tag{13}$$

E becomes a practically continuous quantity.

2.3) In fact, the parameter λ_l was introduced earlier in papers [35]–[43] as a deformation parameter on going from the canonical quantum mechanics to the quantum mechanics at Planck's scales (early Universe) that is considered to be the quantum mechanics with the fundamental length (QMFL):

$$0 < \alpha_x = l_{min}^2/x^2 \leq 1/4, \tag{14}$$

where x is the measuring scale, $l_{min} \sim l_p$.
The deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition [52]. Obviously, everywhere, apart from the limiting point $\lambda_x = 1$ or $x = l_{min}$, we have

$$\lambda_x = \sqrt{\alpha_x}, \tag{15}$$

From (14) it is seen that at the limiting point $x = l_{min}$ the parameter α_x is not defined due to the appearance of singularity [35]–[43]. But at this point its definition may be extended (regularized).
The parameter α_l has the following clear physical meaning:

$$\alpha_l^{-1} \sim S^{BH}, \tag{16}$$

where

$$S^{BH} = \frac{A}{4l_p^2} \tag{17}$$

is the well-known Bekenstein-Hawking formula for the black hole entropy in the semiclassical approximation [53],[54] for the black-hole event horizon surface A, with the characteristics linear dimension («radius») $R = l$. This is especially obvious in the spherically-symmetric case.

In what follows we use both parameters: λ_x and α_x.
Next we show how the **discrete** parameters $\alpha_x^{\gamma/2}$ (or λ_x^{γ}) are naturally involved into the well-known gravity and cosmology models. We try to clarify why at low energies their role is so insignificant and can be neglected, whereas the theory can be considered practically (i.e.**to a high degree of accuracy**) continuous.
Note that at low energies the flat metric $(1, -1, -1, -1)$, with a high degree of accuracy, gives the ansatz:

$$g_{\mu\nu}(x) = (\pm\delta_{\mu\nu}\exp(\pm\lambda_l^{\gamma})),$$

where the sign + before $\delta_{\mu\nu}$ corresponds to the case $\mu = \nu = 0$ only.

3 Some Significant Examples

In this section we give some important illustrations of the gravity and cosmology models within the scope of the paradigm stated in this work.

3.1 Static Spherically-Symmetric Space-Time with Horizon

Gravity and thermodynamics of horizon spaces and their interrelations are currently most actively studied [55]–[67]. Let us consider a relatively simple illustration – the case of a static spherically-symmetric horizon in space-time, the horizon being described by the metric

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2. \tag{18}$$

The horizon location will be given by a simple zero of the function $f(r)$, at the radius $r = a$.
This case is studied in detail by T.Padmanabhan in his works [55, 66] and by the author of this paper in [68]. We use the notation system of [66]. Let, for simplicity, the space be denoted as $\mathcal{H}$.
It is known that for horizon spaces one can introduce the temperature that can be identified with an analytic continuation to imaginary time. In the

case under consideration ([66], eq.(116))

$$k_B T = \frac{\hbar c f'(a)}{4\pi}. \tag{19}$$

Therewith, the condition $f(a) = 0$ and $f'(a) \neq 0$ must be fulfilled.
Then at the horizon $r = a$ Einstein's field equations

$$\frac{c^4}{G}\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right] = 4\pi P a^2 \tag{20}$$

where $P = T_r^r$ is the trace of the momentum-energy tensor and radial pressure.
Now we proceed to the variables ≪α≫ from the preceding section (14) to consider (20)in a new notation, expressing a in terms of the corresponding deformation parameter α. In what follows we omit the subscript in formula (14) of α_x, where the context implies which index is the case. In particular, here we use α instead of α_a. Then we have

$$a = l_{min}\alpha^{-1/2}. \tag{21}$$

Therefore,

$$f'(a) = -2l_{min}^{-1}\alpha^{3/2}f'(\alpha). \tag{22}$$

Substituting this into (20) we obtain in the considered case of Einstein's equations in the ≪α–representation≫ the following [68]:

$$\frac{c^4}{G}(-\alpha f'(\alpha) - \frac{1}{2}) = 4\pi P \alpha^{-1} l_{min}^2. \tag{23}$$

Multiplying the left- and right-hand sides of the last equation by α, we get

$$\frac{c^4}{G}(-f'(\alpha)\alpha^2 - \frac{1}{2}\alpha) = 4\pi P l_{min}^2. \tag{24}$$

L.h.s. of (24) is dependent on α. Because of this, r.h.s. of (24) must be dependent on α as well, i. e. $P = P(\alpha)$, i.e

$$\frac{c^4}{G}(-f'(\alpha)\alpha^2 - \frac{1}{2}\alpha) = 4\pi P(\alpha) l_{min}^2. \tag{25}$$

Note that in this specific case the parameter α within constant factors is coincident with the Gaussian curvature K_a [69] corresponding to a:

$$\frac{l_{min}^2}{a^2} = l_{min}^2 K_a. \qquad (26)$$

Substituting r.h.s of (26) into (25), we obtain the Einstein equation on horizon, in this case in terms of the Gaussian curvature

$$\frac{c^4}{G}(-f'(K_a)K_a^2 - \frac{1}{2}K_a) = 4\pi.P(K_a). \qquad (27)$$

This means that up to the constants

$$-f'(K_a)K_a^2 - \frac{1}{2}K_a = P(K_a), \qquad (28)$$

i.e. the Gaussian curvature K_a is a solution of Einstein equations in this case.
Then we examine different cases of the solution (28) with due regard for considerations of Section 2.

3.1.1) First, let us assume that $a \gg l_{min}$. As, according to Section 2, the radius a is quantized, we have $a = N_a l_{min}$ with the natural number $N_a \gg 1$. Then it is clear that the Gaussian curvature $K_a = 1/a^2 \approx 0$ takes a(nonuniform)discrete series of values close to zero, and, within the factor $1/l_{min}^2$, this series represents inverse squares of natural numbers

$$(K_a) = (\frac{1}{N_a^2}, \frac{1}{(N_a \pm 1)^2}, \frac{1}{(N_a \pm 2)^2}, ...). \qquad (29)$$

Let us return to formula (7) in Section 2 for $l = a$

$$|((\widetilde{\delta} a)_\gamma)_{min}| = \beta N_a l_{min} N_a^{-\gamma} = \beta N_a^{1-\gamma} l_{min}, \qquad (30)$$

where β in this case contains the proportionality factor that relates l_{min} and l_P.
Then, according to Section 2, $a_{\pm 1}$ is a measurable value of the radius r following after a, and we have

$$(a_{\pm 1})_\gamma \equiv a \pm ((\widetilde{\delta} a)_\gamma)_{min} = a \pm \beta N_a^{1-\gamma} l_{min} = N_a(1 \pm \beta N_a^{-\gamma}) l_{min}. \qquad (31)$$

But, as $N_a \gg 1$, for sufficiently large N_a and fixed γ, the bracketed expression in r.h.s. (31) is close to 1:

$$1 \pm \beta N_a^{-\gamma} \approx 1. \tag{32}$$

Obviously, we get

$$\lim_{N_a \to \infty} (1 \pm \beta N_a^{-\gamma}) \to 1. \tag{33}$$

As a result, the Gaussian curvature $K_{a_{\pm 1}}$ corresponding to $r = a_{\pm 1}$

$$K_{a_{\pm 1}} = 1/a_{\pm 1}^2 \propto \frac{1}{N_a^2 (1 \pm \beta N_a^{-\gamma})^2} \tag{34}$$

in the case under study is only slightly different from K_a.
And this is the case for sufficiently large values of N_a, for any value of the parameter γ , for $\gamma = 1$ as well, corresponding to the absolute minimum of fluctuations $\approx l_{min}$,or more precisely – to βl_{min}. However, as all the quantities of the length dimension are quantized and the factor β is on the order of 1, actually we have $\beta = 1$.
Because of this, provided the minimal length is involved, l_{min} (7) is read as

$$|(\widetilde{\delta l})_1|_{min} = l_{min}. \tag{35}$$

But, according to (9), $l_{min} = \xi l_P$ is on the order of Planck's length, and it is clear that the fluctuation $|(\widetilde{\delta l})_1|_{min}$ corresponds to Planck's energies and Planck's scales. The Gaussian curvature K_a, due to its smallness ($K_a \ll 1$ up to the constant factor l_{min}^{-2}) and smooth variations independent of γ (formulas (31)–(34)), is **insensitive** to the differences between various values of γ.
Consequently, for sufficiently small Gaussian curvature K_a we can take any parameter from the interval $0 < \gamma \leq 1$ as γ.
It is obvious that the case $\gamma = 1$, i. e. $|(\widetilde{\delta l})_1|_{min} = l_{min}$, is associated with infinitely small variations da of the radius r in the Riemannian geometry.
Since then K_a is varying practically continuously, in terms of K_a up to the constant factor we can obtain the following:

$$d[L(K_a)] = d[P(K_a)], \tag{36}$$

Where have

$$L(K_a) = -f'(K_a)K_a^2 - \frac{1}{2}K_a, \tag{37}$$

i. e. l.h.s of (27) (or (28)).
But in fact, as in this case the energies are low, it is more correct to consider

$$L((K_{a_{\pm 1}})_\gamma) - L(K_a) = [P(K_{a_{\pm 1}})_\gamma] - [P(K_a)] \equiv F_\gamma[P(K_a)], \tag{38}$$

where $\gamma < 1$,rather than (36).
In view of the foregoing arguments (3.1.1), the difference between (38) and (36)is insignificant and it is perfectly correct to use (36)instead of (38).
In [66] it is shown that the Einstein Equation for horizon spaces in the differential form may be written as a thermodynamic identity (the first principle of thermodynamics) ([66], formula (119)):

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T} \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4}4\pi a^2\right)}_{dS} \underbrace{-\frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{Pd\left(\frac{4\pi}{3}a^3\right)}_{P\,dV}. \tag{39}$$

However, this is questionable on account of the existing minimal length l_{min}. As the quantity l_{min} is fixed, it is obvious that ≪dS≫ and ≪dV≫ in (39)will be growing as a and a^2, respectively. And at low energies, i.e. for large values of $a \gg l_{min}$, this naturally leads to infinitely large rather than infinitesimal values.
We will revert to this problem in Section 4 in connection with analysis of the renowned work by E. Verlinde [72] ≪Entropic Approach to Gravity≫ within the scope of the minimal length, l_{min} theory.
3.1.2) Now we consider the opposite case or the transition to the **ultraviolet limit**

$$a \rightarrow l_{min} = \kappa l_{min}, \tag{40}$$

i.e.

$$a = \kappa l_{min}. \tag{41}$$

Here κ is on the order of 1.
Taking into consideration point 2.1) of Section 2 stating that in this case

models for different values of the parameter γ are coincident, by formula (35) for any γ we have

$$|(\widetilde{\delta l})_{\gamma}|_{min}| = (\widetilde{\delta l})_{1}|_{min} = l_{min}. \qquad (42)$$

But in this case the Gaussian curvature K_a is not a «small value» continuously dependent on a , taking, according to (34), a discrete series of values $K_a, K_{a\pm1}, K_{a\pm2}, ...$
Yet (20), similar to (27) ((28)), is valid in the semiclassical approximation only, i.e. at **low energies**.
Then in accordance with the above arguments, the limiting transition to **high energies**(40) gives a discrete chain of equations or a single equation with a discrete set of solutions as follows:

$$-f'(K_a)K_a^2 - \frac{1}{2}K_a = \Theta(K_a);$$

$$-f'(K_{a\pm1})K_{a\pm1}^2 - \frac{1}{2}K_{a\pm1} = \Theta(K_{a\pm1});$$

and so on. Here $\Theta(K_a)$ – some function that in the limiting transition to low energies must reproduce the low-energy result to a high degree of accuracy, i.e. $P(K_a)$ appears for $a \gg l_{min}$ from formula (28)

$$\lim_{K_a \to 0} \Theta(K_a)) = P(K_a). \qquad (43)$$

In general, $\Theta(K_a)$ may lack coincidence with the high-energy limit of the momentum-energy tensor trace(if any):

$$\lim_{a \to l_{min}} P(K_a). \qquad (44)$$

At the same time, when we naturally assume that the Static Spherically-Symmetric Horizon Space-Time with the radius of several Planck's units (41) is nothing else but a micro black hole, then the high-energy limit (44) is existing and the replacement of $\Theta(K_a)$ by $P(K_a)$ in r.h.s. of the foregoing equations is possible to give a hypothetical gravitational equation for the

event horizon micro black hole. But a question arises, for which values of the parameter a (41) (or K_a) this is valid and what is a minimal value of the parameter $\gamma = \gamma(a)$ in this case?
In all the cases under study, 3.1.1) and 3.1.2), the deformation parameter α_a (14)(λ_a(15))is, within the constant factor, coincident with the Gaussian curvature K_a (respectively $\sqrt{K_a}$) that is in essence continuous in the low-energy case and discrete in the high-energy case.

3.2 Heuristic Markov's Model

This heuristic model was introduced in the work [70] at the early eighties of the last century. In [70], it is assumed that ≪by the universal decree of nature a quantity of the material density ϱ is always bounded by its upper value given by the expression that is composed of fundamental constants≫ ([70], p.214):

$$\varrho \leq \varrho_p = \frac{c^5}{G^2 \hbar}, \tag{45}$$

with ϱ_p as ≪Planck's density≫.
Then the quantity

$$\wp_\varrho = \varrho / \varrho_p \leq 1 \tag{46}$$

is the **deformation parameter** as it is used in [70] to construct the following **of Einstein's equations deformation or $\wp_\varrho$-deformation** ([70],formula (2)):

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \frac{8\pi G}{c^4} T_\mu^\nu (1 - \wp_\varrho^2)^n - \Lambda \wp_\varrho^{2n} \delta_\mu^\nu, \tag{47}$$

where $n \geq 1/2$, T_μ^ν–energy-momentum tensor, Λ– cosmological constant.
The case of the parameter $\wp_\varrho \ll 1$ or $\varrho \ll \varrho_p$ correlates with the classical Einstein equation, and the case when $\wp_\varrho = 1$ – with the de Sitter Universe. In this way (47) may be considered as $\wp_\varrho$-deformation of the General Relativity.
As shown in[68], $\wp_\varrho$-of Einstein's equations deformation (47) is nothing else but α-deformation of GR for the parameter α_l at $x = l$ from (14).
If $\varrho = \varrho_l$ is the average material density for the Universe of the characteristic

linear dimension l, i.e. of the volume $V \propto l^3$, we have

$$\wp_{l,\varrho} = \frac{\varrho_l}{\varrho_p} \propto \alpha_l^2 = \omega\alpha_l^2, \tag{48}$$

where ω is some computable factor.
Then it is clear that α_l-representation (47) is of the form

$$R_\mu^\nu - \frac{1}{2}R\delta_\mu^\nu = \frac{8\pi G}{c^4}T_\mu^\nu(1-\omega^2\alpha_l^4)^n - \Lambda\omega^{2n}\alpha_l^{4n}\delta_\mu^\nu, \tag{49}$$

or in the general form we have

$$R_\mu^\nu - \frac{1}{2}R\delta_\mu^\nu = \frac{8\pi G}{c^4}T_\mu^\nu(\alpha_l) - \Lambda(\alpha_l)\delta_\mu^\nu. \tag{50}$$

But, as r.h.s. of (50) is dependent on α_l of any value and particularly in the case $\alpha_l \ll 1$, i.e. at $l \gg l_{min}$, l.h.s of (50) is also dependent on α_l of any value and (50) may be written as

$$R_\mu^\nu(\alpha_l) - \frac{1}{2}R(\alpha_l)\delta_\mu^\nu = \frac{8\pi G}{c^4}T_\mu^\nu(\alpha_l) - \Lambda(\alpha_l)\delta_\mu^\nu. \tag{51}$$

Thus, in this specific case we obtain the explicit dependence of GR on the available energies $E \sim 1/l$, that is insignificant at low energies or for $l \gg l_{min}$ and, on the contrary, significant at high energies, $l \rightarrow l_{min}$.

3.2.1) At low energies we get the generalization of point 3.1.1) from the previous subsection (precisely at $\mu = \nu$), the only difference being the fact that in this case there is no Gaussian curvature and we have the dependence on α_l instead.
And all the relations (30)–(33) remain valid when a is replaced by l.
In this way, similar to 3.1.1), we get a ≪**nearly continuous theory**≫ with the slowly (smoothly) varying parameter $\alpha_{l(t)}$, where t – time.

3.2.2) Clearly, at high energies the parameter $\alpha_{l(t)}$ is discrete and for the limiting value $\alpha_{l(t)} = 1/4$ (and hence $\lambda_{l(t)} = 1/2$) or close to this value we get a discrete series of equations of the form (50)(or a single equation of this form met by a discrete series of solutions) corresponding to $\alpha_{l(t)} = 1/4; 1/16; 1/36; ...$
As this takes place, $T_\mu^\nu(\alpha_l) \approx 0$ and in both cases, 3.2.1) and 3.2.2), $\Lambda(\alpha_l)$ is not longer a cosmological constant, being a dynamical cosmological term.

3.3 Standard Cosmological Model and Minimal Length

Quite naturally and trivially, the arguments of subsection 3.1 are applicable to the Standard Cosmological Model (SCM).

Let us examine this model within the scope of the considerations given in the present work. We use the notations of [71]. Then for the **Robertson-Walker** metrics we have

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{52}$$

with the spatial curvature κ, where $\kappa = 0, -1, 1$ for the case of flat, open, and close Universe, respectively.

For the homogeneous and isotropic case with the energy-momentum tensor of the form

$$T^{\mu}_{\nu} = \mathrm{Diag}\left(-\rho(t), p(t), p(t), p(t)\right), \tag{53}$$

where ρ corresponds to the energy density and P to the pressure **Friedmann-Lemaître equations**, we have the following:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2}, \tag{54}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{55}$$

As, by definition, $H \equiv \dot{a}/a$ is the *Hubble parameter*, and radius of the apparent (Hubble) horizon is equal to

$$R_H = \frac{1}{\sqrt{H^2 + \kappa/a^2}}, \tag{56}$$

then the equation (55) may be as follows:

$$\frac{1}{R_H^2} = \frac{8\pi G\rho}{3}. \tag{57}$$

In the flat case $\kappa = 0$ and at low energies $a(t) \propto t^{\tau}, 0 < \tau < 1$ we have $R_H \propto t$, $(\ddot{a}/a) \propto t^{-2}$ and (55), according to (57), takes the form

$$\frac{\eta}{R_H^2} = -\frac{4\pi G}{3}\left(\rho + 3p\right) = -\frac{1}{2R_H^2} - 4\pi Gp \tag{58}$$

or

$$\frac{2\eta+1}{R_H^2}=-8\pi Gp, \tag{59}$$

where $\eta-$ corresponding proportionality factor.
It is obvious that for a sufficiently large radius of the Hubble horizon

$$R_H \gg l_{min} \tag{60}$$

equations (54),(55), or what is the same

$$\frac{1}{R_H^2}=\frac{8\pi G\rho}{3}, \tag{61}$$

$$\frac{2\eta+1}{R_H^2}=-8\pi Gp, \tag{62}$$

are actually insensitive to the magnitude of variations in R_H, meaning that upon replacement of $a \rightarrow R_H$ and hence $N_a \rightarrow N_{R_H}$ formulae (30)–(33) are valid and in this case, despite the presence of the minimal length, in essence we derive a continuous theory that is almost independent of the parameter γ. Specifically, we can differentiate in time (61) and (62).

4 Some Comments and General Considerations

4.1. So, as demonstrated in the previous Section for the particular cases, provided a theory involves the minimal length $l_{min} \propto l_P$, gravity is almost independent of the parameters associated with this length, specifically α_l and γ (and hence λ_l and γ), i.e. the dependence is weak, and so the theory is practically continuous. This stems from the fact that these parameters are very small due to remoteness of the energies characterizing them from the Planck energies and almost **insensitive** to the corresponding change in measuring scales.
Despite a **discrete** nature of the theory owing to the existence of l_{min}, to a high degree of accuracy we can use infinitesimal variations of dx_μ, coincident in the case under study with l_{min} and t_{min}. In this way in the cases

considered in Section 3 the **Conformity Principle** stating that (*on going to low energies the known theory (in particular GR) must be reproduced to a high degree of accuracy, at least its experimentally verified part*) holds **to a high accuracy**.
Still it is clear that, as formally GR has no additional parameters associated with l_{min} and the low-energy (for now hypothetical variant of the minimal length theory denoted as $Grav^{l_{min}}$ has such parameters, there is also the **high accuracy** limit indicated above. This limit in every case determines the ≪gap≫ between GR and $Grav^{l_{min}}$. Evaluation of this gap is a real challenge for those trying to construct a unified theory at all energy levels.
As noted in 3.1.2 and 3.2.2, for high energies, i.e. for $l \rightarrow l_{min}$, (or what is the same $\lambda_l \rightarrow 1, \gamma \rightarrow 1$) a discrete chain of equations (or a single equation with a discrete set of solutions) is derived that is numbered by inverse squares of the integers $1; 1/4; 1/9; ....$ to represent the parameter λ_l^2 at high (Planck's) energies.

4.2. We have used GR to demonstrate that the above models 3.1–3.3 at low energies are actually insensitive to variations of the discrete parameters $(\alpha_l (or \lambda_l), \gamma)$ associated with the minimal length. Of course, it is more correct to use $Grav^{l_{min}}$ and to compare the obtained results with GR. But, as yet there is no $Grav^{l_{min}}$, it is connived that at low energies GR and $Grav^{l_{min}}$ differ insignificantly and the indicated parameters, provided l_{min} is involved, are introduced into GR similarly to $Grav^{l_{min}}$.

4.3. It is easily seen that the ≪Entropic Approach to Gravity≫ [72] in the present formalism is invalid within the scope of the minimal length theory. In fact, the ≪main instrument≫ in [72] is a formula for the infinitesimal variation dN in the bit numbers N on the holographic screen $\mathcal{S}$ with the radius R and with the surface area A ([72],formula (4.18)):

$$dN = \frac{c^3}{G\hbar} \, dA = \frac{dA}{l_P^2}. \qquad (63)$$

But it is obvious that infinitesimal variations of the screen surface area dA are possible only in a continuous theory involving no l_{min}.
When $l_{min} \propto l_P$ is involved, the minimal variation $\triangle A$ is evidently associ-

ated with a minimal variation in the radius R

$$R \rightarrow R \pm l_{min} \tag{64}$$

, is dependent on R and growing with $\sim R$ for $R \gg l_{min}$

$$\triangle_{\pm} A(R) = A(R \pm l_{min})) - A(R) \propto (\frac{\pm 2R}{l_{min}} + 1) = \pm 2N_R + 1, \tag{65}$$

where $N_R = R/l_{min}$, as indicated above.
So, if l_{min} is involved, formula (4.18) from [72] has no sense similar to other formulae derived on its basis (4.19),(4.20),(4.22),(5.32)–(5.34), ... in [72] and similar to the derivation method for Einstein's equations proposed in this work.
Proceeding from the principal parameters of this work $\alpha_l (or \lambda_l)$, the fact is obvious and is supported by the formula (16)given in this paper, meaning that

$$\alpha_R^{-1} \sim A, \tag{66}$$

i.e. small variations of α_R (low energies) result in large variations of α_R^{-1}, as indicated by formula (65).
In fact, we have a **no-go theorem**.

5 Conclusion

Thus, provided the main concept of this work is realized, at all energy levels gravity may be governed by **the same set of discrete parameters** which still have different variation rates and differing values in the low- and high-energy regions. The transfer from low energies (GR) to high (Planck) energies (QG) may be schematically represented as

$$GR[k_l \gg 1, \lambda \approx 0, ...] \rightarrow QG[k_l \rightarrow 1, \lambda \rightarrow 1, ...]. \tag{67}$$

However, as noted in [52], in nature the direction was opposite – from high to low energies. Because of this, it seems more natural to consider the transition opposite to (67), in [52] referred to as the ≪**dequantization**≫:

$$QG[k_l \rightarrow 1, \lambda \rightarrow 1, ...] \rightarrow GR[k_l \gg 1, \lambda \approx 0, ...] \tag{68}$$

The parameter set in the left and right sides of (67),(68) is the same. The dots in parenthesis are given for additional parameters which may arise in the process of a theory resolution.
It is important that in this case **gravity could be considered as a single whole without its subdivision into the Classical Gravity (GR) and Quantum Gravity (QG)**.
Of course, the dependence of the principal space-time quantities on the measuring scale l (existing energy E) that is based on STQF and suggested in (6),(7) is very tentative and may vary in the process of the theory evolution. Still, by author's opinion, a set of the principal discrete parameters in (67) will be invariant with respect to these variations.
The primary criterion for resolution of a future theory must be the **Conformity Principle**:
on going to low energies GR must be reproduced to a high degree of accuracy, at least its experimentally verified part.
Thus gravity by this approach for all the energy levels is treated as a single whole (one building), where the descent from the upper levels (steps) to the lower ones by the energy steps is governed by a single discrete parameter λ_l, the step height being steadily reduced as we descend lower and lower, whereas their length $\sim l$ will be ever increasing.

In this context, it has been shown that some models for GR (cosmology) involve the mentioned discrete parameters associated with the minimal length, while at low energies, due to their smallness, a theory is **insensitive** to their variations and may be considered almost continuous, independent of these parameters.

As at low energies $\alpha_l(\lambda_l)$–small parameter, the gap between GR and a hypothetical minimal length theory $Grav^{l_{min}}$ (mentioned in subsection 4.1) is determined by a series expansion in terms of this parameter close to 0 and by confinement of the leading terms in this series.
As in this case the cosmological term Λ is no longer a constant $\Lambda \neq const$, (and the Bianchi identity $\nabla^{\mu}G_{\mu\nu} \approx 0$ [1] will appear to a high degree of accuracy only in the low-energy limit), this term is dependent on $\alpha_l(\lambda_l)$ and

we have [73],[68] with the known quantum field theory

$$\Lambda(\alpha) \propto (\alpha^2 + \eta_1\alpha^2 + ...)\Lambda_p, \tag{69}$$

and, provided the holographic principle is valid, we get [74]–[76]

$$\Lambda^{Hol}(\alpha) \propto (\alpha + \xi_1\alpha^2 + ...)\Lambda_p, \tag{70}$$

where Λ_p –cosmological term at Planck's scales.

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