First Draft

Lorentz Symmetry from Multifractal Scaling

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Abstract

We show that relativistic invariance is encoded in the multifractal structure of the Standard Model near the electroweak scale. The approximate scale invariance of this structure accounts for the flavor hierarchy and chiral symmetry breaking in the electroweak sector. Surprisingly, it also accounts for breaking of conformal symmetry in General Relativity and the emergence of a non-vanishing cosmological constant.

Key words: minimal fractal manifold, multifractal scaling, scale invariance, Lorentz group, conformal symmetry, cosmological constant.

Introduction

It is well-known that invariance under the Lorentz group (LG) is a fundamental requirement of both Quantum Field Theory (QFT) and the Standard Model (SM). The relativity principle demands the equivalence of inertial frames of reference under Lorentz transformations including rotations, boosts and space-time inversions. The Poincaré group is a natural extension of LG and accommodates translations in flat space-time. Lorentz transformations leave the 4-vector inner product invariant and enable the definition of generators associated with the conserved Noether charges of LG.

The object of this work is to explore the unforeseen link between LG and the multifractal structure of SM near the infrared regime set by the electroweak scale ($M_{EW} \approx 250\,\text{GeV}$). Specifically, exploiting the *minimal fractal manifold* geometry (MFM) of space-time near M_{EW} [ ], we show how LG may be naturally mapped to continuous and scale-dependent transformations of four space-time dimensions described by $\varepsilon = 4 - D << 1$.

The paper is organized in the following way: The first section develops the formal connection between the multifractal description of SM near M_{EW} and the group of Lorentz transformations. Next section uses the concept of *dimensional flow* to uncover the two Casimir invariants of QFT, namely "mass" and "spin", as well as the full flavor hierarchy of the SM. Breaking of exact scale invariance near M_{EW} naturally explains the violation of space and time inversion in electroweak channels. The third section shows how the large separation of scales induced by the MFM accounts for breaking of conformal symmetry in General Relativity and the emergence of a non-vanishing cosmological constant. Summary and concluding remarks are reported in the last section. Two Appendix sections are also included to make the paper self-contained and accessible to a large audience.

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$$\varepsilon = 4 - D = \sum_{\mu=0}^{3}(1 - d_\mu) = \sum_{\mu=0}^{3}\varepsilon_\mu$$

$$\varepsilon = \frac{m^2}{\Delta_{UV}^2}, \quad \varepsilon_\mu = \frac{m_\mu^2}{\Delta_{UV}^2}$$

$$\varepsilon = \varepsilon_0 + \sum_{i=1}^{3} \varepsilon_i$$

$$\varepsilon_0 = \varepsilon - \sum_{i=1}^{3} \varepsilon_i$$

$$\varepsilon = O\left(\frac{m^2}{\Lambda_{UV}^2}\right)$$

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$$P'^{\mu} = \Lambda_{\nu}^{\mu} P^{\nu}$$

$$P' = \Lambda P \Rightarrow (\varepsilon_{\mu}^{'})^{1/2} = \Lambda (\varepsilon_{\mu})^{1/2}$$

$$\Lambda_{\nu}^{\mu} = g_{\nu}^{\mu} + \omega_{\nu}^{\mu}$$

$$g_{\rho\sigma} = g_{\mu\nu} \Lambda_{\rho}^{\mu} \Lambda_{\sigma}^{\nu}$$

$$P^2 = P^{\mu} P_{\mu} = E^2 - \mathbf{P}^2 \Rightarrow -P^2 = \mathbf{P}^2 - E^2$$

$$P^{\mu} P_{\mu} = m^2$$

$$\Lambda = \exp[-\frac{1}{2} \omega_{\mu\nu} J^{\mu\nu}] = \exp[-i\boldsymbol{\theta} \cdot \mathbf{J} + i\boldsymbol{\eta} \cdot \mathbf{K}]$$

$$\Lambda = I - \frac{1}{2} \omega_{\mu\nu} J^{\mu\nu}$$

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$$\Lambda_{cc}^{1/4} = O(\frac{M_{EW}^2}{M_{Pl}}) = O(G_N^{1/2} M_{EW}^2)$$

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