Mach or Higgs? The mechanisms to generate mass

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With the imminent confirmation or disproof of the existence of Higgs boson by LHC experiments it is time to analyze in a non-dogmatic way the proposals to understand the origin of the mass. Here we analyze a new version of Mach's principle according to which gravity is what is really responsible for the generation of mass of all bodies. The condition for this is the existence of an energy distribution represented by the vacuum or the cosmological constant term $\Lambda\, g_{\mu\nu}$. The great novelty of this mechanism is that the gravitational field acts merely as a catalyst, once the final expression of the mass depends neither on the intensity and particular properties of the gravitational field nor on the value of Newton's constant. It was precisely the wrong belief that the value of the mass obtained through any gravitational scheme must depend on Newton's constant which was responsible for not considering gravity as an important actor in the mechanism of generating mass. We review briefly the alternative Higgs mechanism in order to compare both processes.

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I.   INTRODUCTION

In order to become a reliable candidate as a mechanism to generate mass, there are three indispensable conditions that such mechanism has to fulfil, to wit:

- There must exist a universal field that interacts with all kinds of particles;

- This field must be such that its interaction with matter breaks explicitly some symmetry that only massless particles exhibit, e.g. the gauge freedom for vector fields or the chirality for fermions.

- There must exist a free dimensionless parameter such that different bodies can acquire distinct values for their corresponding mass (the spectrum of mass);

There are only two fashionable candidates[21] that fulfill the first condition:

- The gravitational field.

- A scalar field φ;

The Higgs boson φ was postulated by Higgs, Brout and Englert to couple universally with all kinds of matter. However, still to this day there is no evidence of its existence. The other candidate, gravity, is known to couple with all forms of matter and energy and its universality is recognized as a scientific truth. We note that after accepting either one of these two fields as a good candidate that fulfills the first requirement, it is not a hard job to elaborate scenarios such that the other two conditions are satisfied too.

The purpose of these notes is to compare these two mechanisms.

In order to simplify our analysis we will present the generation of mass for the elementary constituents of matter, identified as the basic bricks of all matter. They will be taken as representations of the Lorentz-Poincaré group and we will analyze them as scalar, spinor, vector and tensor fields. We shall see that in both cases the origin of the mass of any body $\mathbb{A}$ depends on its interaction with its surroundings yielding an overall effect (described either as a scalar field – in the case of the Higgs mechanism – or as the metric tensor of the geometry of space-time - in the case of the gravitational origin) on $\mathbb{A}$ which is represented by a distribution of energy given by the form

$$T_{\mu\nu} = \lambda \, g_{\mu\nu} \qquad (1)$$

In the literature concerning General Relativity this form of energy-momentum tensor is attributed to the cosmological constant introduced by Einstein in order to be able to construct a model for the geometry of the universe. In the realm of quantum field theory, such distribution is identified as the vacuum. It is true that if one considers the Machian point of view that the inertia of a body $\mathbb{A}$ depends on the energy distribution of all others bodies in the universe, then λ is to be interpreted as the cosmological constant [12].

The idea of using a scalar field to be at the origin of the mass appeared in the domain of high energy physics and it received the name "Higgs mechanism". For the time being there is no evidence of the existence of such scalar in Nature and huge experiments - the LHC experiment – are at this very moment on the verge of being realized in order to prove that such scalar field exists [1], [17].

On the other hand, the relationship of mass with gravity is a very old one and its deep connection has been emphasized in a qualitative way many times. We will concentrate our analysis only on a particular process that admits a systematic realization and allows for a quantification.

Although the theory of General Relativity may be understood as completely independent from the Machian idea that inertia of a body $\mathbb{A}$ is related to the global distribution of energy of all particles existing in the universe, we must recognize its historical value in the development of the ideology that enabled Einstein to start his journey toward the construction of a theory of gravitation [2].

During the 20th century, the idea of associating the dependence of local characteristics of matter with the global state of the universe came up now and then but without producing any reliable mechanism that could support this proposal [3]. Even the concept of mass – that pervades all gravitational processes – did not find a realization of such dependence on global structure of the universe. On the contrary, the most efficient mechanism and one that has performed an important role in the field of microphysics came from elsewhere, namely the attempt to unify forces of a non-gravitational character, such as long-range electrodynamics with decaying phenomena described by weak interaction. Indeed, the Higgs model produced an efficient scenario for generating mass to the vector bosons [4] that goes in the opposite direction of the proposal of Mach. This mechanism starts with the transformation of a global symmetry into a local one and the corresponding presence of vector gauge fields. Then, a particular form of the dynamics represented by $L_{int}(\varphi)$ of self-interaction of an associated

scalar field in its fundamental state represented by an energy-momentum tensor given by $T_{\mu\nu} = L_{int}(\varphi_0)\, g_{\mu\nu}$ appears as the vehicle which provides mass to the gauge fields.

Recently a new mechanism for generation of mass that is a realization of Mach's idea was proposed [5]. The strategy used is to couple non-minimally the field (scalar, spinor [6], vector [7] and tensor) to gravity through the presence of terms involving explicitly the curvature of space-time. The distribution of the vacuum energy of the rest-of-the-universe is represented by a cosmological term Λ. The effect of Λ by the intermediary of the dynamics of the metric of space-time in the realm of General Relativity is precisely to give mass to the field. Although this mass depends on the cosmological constant, its value cannot be obtained a priori [8].

II. THE HIGGS PROPOSAL

Consider a theory of a real scalar field φ described by the Lagrangian

$$\mathbb{L} = \frac{1}{2}\, \partial_\mu\varphi\, \partial^\mu\varphi - V(\varphi) \tag{2}$$

where the potential has the form

$$V = \frac{1}{2}\, \mu^2\, \varphi^2 + \frac{\lambda}{4}\, \varphi^4$$

In the homogeneous case, in order to satisfy the equation of motion, the field must be in an extremum of the potential, which is true for two classes of solution: either

$$\varphi = 0$$

or

$$\varphi_0^2 = -\frac{\mu^2}{\lambda}.$$

In order to be a minimum the constant μ^2 must be negative. This is a problem, since it should imply that the mass of the scalar field is imaginary! However, one can avoid this difficulty in the following manner. Let us redefine the field by introducing a new real variable χ :

$$\varphi = \varphi_0 + \chi,$$

where φ_0 is a constant. Substituting this definition on Lagrangian (2) it follows

$$\mathbb{L} = \frac{1}{2}\, \partial_\mu\chi\, \partial^\mu\chi + \mu^2\, \chi^2 - \frac{\lambda}{4}\, \chi^4 - \lambda\, \varphi_0\, \chi^3 + \frac{\mu^4}{4\lambda} \tag{3}$$

This Lagrangian represents a real scalar field χ with real positive mass $m^2 = -\mu^2$ and extra terms of self-interaction. Note that in the Lagrangian it appears a residual constant term representing a background constant negative energy distribution

$$T_{\mu\nu}(residual) = -\frac{\mu^4}{4\lambda}\, g_{\mu\nu}$$

In the realm of high energy physics it is considered that such a term ... " has no physical consequences and can be dropped" [15]. We will come back to this when we analyze its gravitational effects.

Note that now, the potential of field χ takes the form

$$V = m^2\, \chi^2 + \frac{\lambda}{4}\, \chi^4 + \lambda\, \varphi_0\, \chi^3$$

Its minimum occurs for $\chi = 0$. The other two extrema that exist for constant values χ_0 are points of maxima. The expansion of the field must be made (for all calculations) around $\chi = 0$ and not around $\varphi = 0$. The reason is that this last is an unstable point and the series will suffer from convergence. Finally, we note that the actual field χ has a real positive mass.

A.   The case of complex field

Let us now turn to the case of a complex field. The Lagrangian for $\phi = \phi_1 + i\,\phi_2$ is given by

$$\mathbb{L} = \partial_\mu \phi^*\, \partial^\mu \phi - V(\phi^*\, \phi) \tag{4}$$

where the potential has the form

$$V = \mu^2\, \phi^*\, \phi + \lambda\, (\phi^*\, \phi)^2$$

It is convenient to write the field as

$$\phi = \frac{1}{\sqrt{2}}\, (\phi_0 + \chi)\, \exp \frac{i}{\phi_0}\, \theta(x)$$

The Lagrangian then becomes

$$\begin{aligned} \mathbb{L} = {}& \frac{1}{2}\, \partial_\mu \chi\, \partial^\mu \chi + \frac{1}{2}\, \frac{(\phi_0 + \chi)^2}{\phi_0^2}\, \partial_\mu \theta\, \partial^\mu \theta \\ & - \frac{\mu^2}{2}\, (\phi_0 + \chi)^2) - \frac{h}{4}\, (\phi_0 + \chi)^4 \end{aligned} \tag{5}$$

The extremum of the potential occurs for $\phi_0 + \chi = 0$. For $\mu^2 > 0$ this extremum is a minimum.

B.   From global to local symmetry

The theory of the complex field ϕ has a gauge invariance under the constant map

$$\phi' = e^{i\alpha}\,\phi.$$

This means that this transformation occurs everwhere and does not distinguish any point of space-time. If the parameter α becomes space-time dependent the symmetry is broken. In order to restore the symmetry, one can use the freedom of the electromagnetic field A_μ and couple this map with the map

$$A'_\mu = A_\mu - \frac{1}{e}\,\partial_\mu\alpha.$$

In early 1954 this scheme was generalized for more general maps (non-abelian theory) by Yang and Mills for nonlinear fields, called generically gauge fields. It shows immediately that by minimal coupling of the scalar field with a gauge field the symmetry is restored. The modification consists in the passage from a global symmetry (valid for transformations that are the same everywhere) to a local symmetry that depends on the space-time location. A global property turns into a local one. It is like going from a cosmological framework – that deals with the global structure of space-time — to microphysics.

C.   Mass for a vectorial boson

The interaction of the complex field ϕ with a vector W_μ through the substitution of the derivatives of the scalar field $\partial_\mu\phi$ by $(\partial_\mu - ieW_\mu)\,\phi$ using the minimum coupling principle, preserves the gauge invariance when the parameter α becomes a function of space-time $\alpha(x)$. This means that the dynamics is invariant under the map

$$\phi' = \phi\exp i\,\alpha(x)$$

$$W'_\mu = W_\mu + \frac{1}{e}\,\partial_\mu\alpha$$

We set

$$\mathbb{L} = -\frac{1}{4}\,F^{\mu\nu}\,F_{\mu\nu} + (\partial_\mu - ieW_\mu)\,\phi(\partial^\mu + ieW^\mu)\,\phi^* - V \tag{6}$$

Let us re-define the field by

$$\phi = \frac{1}{2}\left(\phi_0 + \chi\right)e^{i\theta/\phi_0}$$

We can use the invariance under the gauge and choose

$$\alpha = -\frac{\theta}{\phi_0}$$

to eliminate θ. The dynamics turn into

$$
\begin{aligned}
\mathbb{L} = {} & -\frac{1}{4}\,F^{\mu\nu}\,F_{\mu\nu} + \frac{e^2\,\phi_0^2}{2}\,W_\nu\,W^\nu \\
& + \frac{1}{2}\,\partial_\mu\chi\,\partial^\mu\chi - V(\chi) \\
& + \left(e^2\,\varphi_0\,\chi + \frac{e^2}{2}\,\chi^2\right)W_\nu\,W^\nu
\end{aligned}
\tag{7}
$$

that represents a massive vector field interacting non-minimally with a real scalar field. Note that one of the degrees of freedom of the theory – represented by the scalar field $\theta(x)$ — was eliminated. Indeed, it was transformed into an extra degree of freedom of the massive vector field (that gained one more degree of liberty going from 2 to 3). The total number we had (two for the field ϕ and two for the massless field W_μ) is preserved. It only changed places. The degree of freedom of θ was conceded to the (now) massive vector boson.

It is not difficult to generalize the above procedure for more than one vector field in such a way that one of them remains massless. This was the procedure for the case of the unified field theory of electro-weak interaction: the intermediary boson gain a mass but the photon remains massless.

D. Mass for a fermion

Let us couple this scalar field with a spinor Ψ through the Lagrangian

$$\mathbb{L} = \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi) + \mathbb{L}_D - h\,\varphi\,\bar{\Psi}\,\Psi \tag{8}$$

where $\mathbb{L}_D$ is Dirac dynamics for massless free field. Making the same replacement we made previously using χ instead of φ this theory becomes

$$\mathbb{L} = \frac{1}{2}\,\partial_\mu\chi\,\partial^\mu\chi - V(\chi) + \mathbb{L}_D - h\left(\varphi_0 + \chi\right)\bar{\Psi}\,\Psi \tag{9}$$

The equation for the spinor field becomes

$$i\gamma^\mu \partial_\mu \Psi - h\,\varphi_0\,\Psi - h\,\chi\,\bar{\Psi}\,\Psi = 0 \tag{10}$$

which represents a spinor field of mass $h\,\varphi_0 > 0$ interacting with a scalar field χ.

III.   WHAT GIVES MASS TO THE SCALAR FIELD THAT GIVES MASS FOR THE VECTOR AND SPINOR FIELDS?

In the preceding sections we described the Higgs model that produced an efficient scenario for generating mass to the vector bosons in the realm of high-energy physics. At its origin appears a process relating the transformation of a global symmetry into a local one and the corresponding presence of vector gauge fields.

This mechanism appeals to the intervention of a scalar field that appears as the vehicle which provides mass to the gauge vector field W_μ. For the mass to be a real and constant value (a different value for each field) the scalar field φ must be in a minimum state of its potential V. This fundamental state of the self-interacting scalar field has an energy distribution given by $T_{\mu\nu} = V(\varphi_0)\,g_{\mu\nu}$. A particular form of self-interaction of the scalar field φ allows the existence of a constant value $V(\varphi_0)$ that is directly related to the mass of W_μ. This scalar field has its own mass, the origin of which remains unclear. In [5] a new mechanism depending on the gravitational interaction, that can provide mass to the scalar field was presented. In these lectures we shall analyze this mechanism.

Although the concept of mass pervades most of all analysis involving gravitational inter-action, it is an uncomfortable situation and still to this day there has been no successful attempt to derive a mechanism through which mass is understood a direct consequence of a dynamical process depending on gravity [10].

The main idea concerning inertia in the realm of gravity according to the origins of General Relativity, goes in the opposite direction to the mechanism that we analyzed in the previous section in the territory of the high-energy physics. Indeed, while the Higgs mechanism explores the reduction of a global symmetry into a local one, Mach's principle suggests a cosmical dependence of local properties, making the origin of the mass of a given body dependent on the structure of the whole universe. In this way, there ought to exist a mechanism by means of which this quantity - mass – depends on the state of the universe.

Let us describe an example of such mechanism in order to see how this vague idea can achieve a qualitative scheme [11].

A. Mass for scalar field: a trivial case

We start by considering Mach principle as the statement according to which the inertial properties of a body $\mathbb{A}$ are determined by the energy-momentum throughout all space. How could we describe such universal state that takes into account the whole contribution of the rest-of-the-universe onto $\mathbb{A}$? There is no simpler way than consider this state as the most homogeneous one and relate it to what Einstein attributed to the cosmological constant or, in modern language, the vacuum of all remaining bodies. This means to describe the energy-momentum distribution of all complementary bodies of $\mathbb{A}$ as

$$T_{\mu\nu} = \lambda\, g_{\mu\nu}$$

Let φ be a massless field the dynamics of which is given by the Lagrangian

$$L_\varphi = \frac{1}{2}\, \partial_\alpha\varphi\, \partial^\alpha\varphi$$

In the framework of General Relativity its gravitational interaction is given by the Lagrangian

$$L = \frac{1}{\kappa_0}\, R + \frac{1}{2}\, \partial_\alpha\varphi\, \partial^\alpha\varphi + B(\varphi)\, R - \frac{\lambda}{\kappa_0} \tag{11}$$

where for the time being the dependence of B on the scalar field is not fixed. This dynamics represents a scalar field non-minimally coupled to gravity. The cosmological constant is added by the reasons presented above and represents the influence of the rest-of-the-universe on φ. We shall see that λ is the real responsible to provide mass for the scalar field. This means that if we set $\lambda = 0$ the mass of the scalar field should vanish.

Independent variation of φ and $g_{\mu\nu}$ yields

$$\Box\varphi - R\, B' = 0 \tag{12}$$

$$\alpha_0\, (R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu}) = -T_{\mu\nu} \tag{13}$$

where we set $\alpha_0 \equiv 2/\kappa_0$ and $B' \equiv \partial B/\partial\varphi$. The energy-momentum tensor is given by

$$
\begin{aligned}
T_{\mu\nu} =\; & \partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}\,\partial_\alpha\varphi\,\partial^\alpha\varphi\,g_{\mu\nu} \\
& + 2B\,(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) \\
& + 2\nabla_\mu\nabla_\nu B - 2\Box B\,g_{\mu\nu} + \frac{\lambda}{\kappa_0}\,g_{\mu\nu}
\end{aligned}
\tag{14}
$$

Taking the trace of equation (55) we obtain

$$
(\alpha_0 + 2B)\,R = -\,\partial_\alpha\varphi\,\partial^\alpha\varphi - 6\Box B + \frac{4\lambda}{\kappa_0}
\tag{15}
$$

Inserting this result on the equation (46) yields

$$
\Box\varphi + \mathbb{Z} = 0
\tag{16}
$$

where

$$
\mathbb{Z} \equiv \frac{B'}{\alpha_0 + 2B}\left(\partial_\alpha\varphi\,\partial^\alpha\varphi + 6\Box B - \frac{4\lambda}{\kappa_0}\right)
$$

or, equivalently,

$$
\mathbb{Z} = \frac{B'}{\alpha_0 + 2B}\left(\partial_\alpha\varphi\,\partial^\alpha\varphi(1 + 6B'') + 6\,B'\,\Box\varphi - \frac{4\lambda}{\kappa_0}\right)
$$

Therefore, the scalar field acquires an effective self-interaction through the non-minimal coupling with the gravitational field. At this stage it is worth to select among all possible candidates of B a particular one that makes the factor on the gradient of the field to disappear in the expression of $\mathbb{Z}$ by setting

$$
B = a + b\,\varphi - \frac{1}{12}\,\varphi^2
$$

where a and b are arbitrary parameters. The quantity a makes only a re-normalization of the constant $1/\kappa_0$ and parameter b is responsible for distinguishing different masses for different fields. Making a translation on the field

$$
\Phi = -\varphi + 6b
$$

it follows

$$
\Box\Phi + \mu^2\,\Phi = 0
\tag{17}
$$

where

$$
\mu^2 = \frac{2\lambda}{3}\,\frac{\kappa_{ren}}{\kappa_0}.
\tag{18}
$$

where

$$\kappa_{ren} = \frac{1}{\alpha_0 + 2a + 6b^2}$$

Thus as a result of the above process the scalar field acquires a mass μ that depends on λ. If λ vanishes then the mass of the field vanishes. The net effect of the non-minimal coupling of gravity with the scalar field corresponds to a specific self-interaction of the scalar field. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe in the state in which all existing matter is on the corresponding vacuum. The values of different masses for different fields is contemplated in the parameter b.

B. Mass for scalar field-II

Let us now analyze a more general scenario to provide mass to a scalar field. We start from the Lagrangian that describes a massless field φ that is

$$L_\varphi = \frac{1}{2} \partial_\alpha \varphi \, \partial^\alpha \varphi$$

The gravitational interaction yields the modified Lagrangian

$$L = \frac{1}{\kappa} R + \frac{1}{2} W(\varphi) \, \partial_\alpha \varphi \, \partial^\alpha \varphi + B(\varphi) \, R - \frac{1}{\kappa} \Lambda \tag{19}$$

where for the time being the dependence of B and W on the scalar field is not fixed. We set $\hbar = c = 1$.

This dynamics represents a scalar field coupled non-minimally with gravity. There is no direct interaction between φ and the rest-of-the-universe (ROTU), except through the intermediary of gravity described by a cosmological constant Λ. Thus Λ represents the whole influence of the ROTU on φ.

Independent variation of φ and $g_{\mu\nu}$ yields

$$W \, \Box \varphi + \frac{1}{2} W' \, \partial_\alpha \varphi \, \partial^\alpha \varphi - B' \, R = 0 \tag{20}$$

$$\alpha_0 \left(R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \right) = -T_{\mu\nu} \tag{21}$$

where $\alpha_0 \equiv 2/\kappa$ and $B' \equiv \partial B/\partial \varphi$. The energy-momentum tensor defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta g^{\mu\nu}}$$

is given by

$$
\begin{aligned}
T_{\mu\nu} &= W\,\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}\,W\,\partial_\alpha\varphi\,\partial^\alpha\varphi\,g_{\mu\nu} \\
&+ 2B\,(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) \\
&+ 2\nabla_\mu\nabla_\nu B - 2\Box B\,g_{\mu\nu} + \frac{1}{\kappa}\,\Lambda\,g_{\mu\nu}
\end{aligned}
\tag{22}
$$

Taking the trace of equation (21) we obtain

$$(\alpha_0 + 2B)\,R = -\,\partial_\alpha\varphi\,\partial^\alpha\varphi\,(W + 6\,B'') - 6B'\,\Box\varphi + 4\,\frac{\Lambda}{\kappa} \tag{23}$$

where we used that $\Box B = B'\,\Box\varphi + B''\,\partial_\alpha\varphi\,\partial^\alpha\varphi$.

Inserting this result back on the equation (20) yields

$$\mathbb{M}\,\Box\varphi + \mathbb{N}\,\partial_\alpha\varphi\,\partial^\alpha\varphi - \mathbb{Q} = 0 \tag{24}$$

where

$$\mathbb{M} \equiv W + \frac{6(B')^2}{\alpha_0 + 2B}$$

$$\mathbb{N} \equiv \frac{1}{2}\,W' + \frac{B'\,(W + 6B'')}{\alpha_0 + 2B}$$

$$\mathbb{Q} = \frac{4\,\Lambda\,B'}{\kappa\,(\alpha_0 + 2B)}$$

Thus, through the non-minimal coupling with the gravitational field the scalar field acquires an effective self-interaction. At this point it is worth to select among all possible candidates of B and W particular ones that makes the factor on the gradient of the field to disappear on the equation of motion by setting $\mathbb{N} = 0$. This condition will give W as a function of B :

$$W = \frac{2q - 6(B')^2}{\alpha_0 + 2B} \tag{25}$$

where q is a constant. Inserting this result into the equation (24) yields

$$\Box\varphi - \frac{2\,\Lambda}{q\,\kappa}\,B' = 0. \tag{26}$$

At this point one is led to set

$$B = -\frac{\beta}{4}\,\varphi^2$$

to obtain

$$\Box\varphi + \mu^2\,\varphi = 0 \tag{27}$$

where

$$\mu^2 \equiv \frac{\beta\,\Lambda}{q\,\kappa} \tag{28}$$

For the function W we obtain

$$W = \frac{2\,q - 3\,\beta^2\,\varphi^2}{2\,\alpha_0 - \beta\varphi^2}$$

One should set $q = \alpha_0$ in order to obtain the standard dynamics in case β vanishes. Using units were $\hbar = 1 = c$ we write

$$\mathbb{L} = \frac{1}{\kappa}\,R + \frac{2\,q - 3\,\beta^2\,\varphi^2}{2\,(2\,\alpha_0 - \beta\varphi^2)}\,\partial_\alpha\varphi\,\partial^\alpha\varphi - \frac{1}{4}\,\beta\,\varphi^2\,R - \frac{\Lambda}{\kappa}$$

Thus as a result of the gravitational interaction the scalar field acquires a mass μ that depends on the constant β and on the existence of Λ :

$$\mu^2 = \frac{\beta\,\Lambda}{2} \tag{29}$$

If Λ vanishes then the mass of the field vanishes. The net effect of the non-minimal coupling of gravity with the scalar field corresponds to a specific self-interaction of the scalar field. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe — represented by the cosmological constant — in the state in which all existing matter is on the corresponding vacuum. The values of different masses for different fields is contemplated in the parameter β.

C.   Re-normalization of the mass

The effect of the rest-of-the-universe on a massive scalar field can be analyzed through the same lines as above. Indeed, let us consider the case in which there is a potential $V(\varphi)$

$$L = \frac{1}{\kappa}\,R + \frac{W}{2}\,\partial_\alpha\varphi\,\partial^\alpha\varphi + B(\varphi)\,R - V(\varphi) - \frac{\Lambda}{\kappa} \tag{30}$$

The equation for the scalar field is given by

$$W\,\Box\varphi + \frac{1}{2}\,W'\,\partial_\alpha\varphi\,\partial^\alpha\,\varphi - B'\,R + V' = 0 \tag{31}$$

Use the equation for the metric to obtain the scalar of curvature in terms of the field and Λ. It then follows that terms in $\partial_\alpha \varphi \, \partial^\alpha \varphi$ are absent if we set

$$W = \frac{2q - 6 \, (B')^2}{\alpha_0 + 2B}$$

where q is a constant. For the case in which $B = -\beta \, \varphi^2 / 4$ and for the potential

$$V = \frac{\mu_0}{2} \, \varphi^2$$

and choosing $q = 1/\kappa$ (in order to obtain the standard equation of the scalar field in case $B = 0$) yields

$$\Box \varphi + (\mu_0^2 + \beta \, \Lambda) \, \varphi + \frac{\beta \, \mu_0^2}{4} \, \kappa \, \varphi^3 = 0 \tag{32}$$

This dynamics is equivalent to the case in which the scalar field shows an effective potential (in absence of gravity) of the form

$$V_{eff} = (\mu_0^2 + \beta \, \Lambda) \, \frac{\varphi^2}{2} + \frac{\beta \, \mu_0^2 \, \kappa}{16} \, \varphi^4$$

Thus the net effect of the gravitational interaction for the dynamics driven by (30) is to re-normalize the mass from the bare value μ_0 to the value

$$\mu^2 = \mu_0^2 + \beta \, \Lambda.$$

We can then contemplate the possibility that all bodies represented by a scalar field could have the same bare mass and as a consequence of gravitational interaction acquires a split into different values characterized by the different values of β. This result is not exclusive of the scalar field but is valid for any field.

IV.   THE CASE OF FERMIONS

Let us now turn our attention to the case of fermions. The massless theory for a spinor field is given by Dirac equation:

$$i\gamma^\mu \partial_\mu \, \Psi = 0 \tag{33}$$

This equation is invariant under γ^5 transformation. In order to have mass for the fermion this symmetry must be broken. Who is the responsible for this?

Gravity breaks the symmetry

Electrodynamics appears in gauge theory as a mechanism that preserves a symmetry when one pass from a global transformation to a local one (space-time dependent map). Nothing similar with gravity. On the contrary, in the generation of mass through the mechanism that we are analyzing here, gravity is the responsible to break the symmetry. In the framework of General Relativity the gravitational interaction of the fermion is driven by the Lagrangian

$$
\begin{aligned}
L \; = \; & \frac{i\,\hbar\,c}{2}\bar{\Psi}\gamma^{\mu}\nabla_{\mu}\Psi - \frac{i}{2}\nabla_{\mu}\bar{\Psi}\gamma^{\mu}\Psi \\
+ \; & \frac{1}{\kappa}R + V(\Phi)\,R - \frac{1}{\kappa}\Lambda \\
+ \; & L_{CT}
\end{aligned}
\tag{34}
$$

where the non-minimal coupling of the spinor field with gravity is contained in the term $V(\Phi)$ that depends on the scalar

$$
\Phi \equiv \bar{\Psi}\,\Psi
$$

which preserves the gauge invariance of the theory under the map $\Psi \rightarrow \exp(i\,\theta)\,\Psi$. Note that the dependence on Φ on the dynamics of Ψ breaks the chiral invariance of the mass-less fermion, a condition that is necessary for a mass to appear.

For the time being the dependence of V on Φ is not fixed. We have added a counter-term L_{CT} for reasons that will be clear later on. On the other hand, the form of the counter-term should be guessed, from the previous analysis that we made for the scalar case, that is we set

$$
L_{CT} = H(\Phi)\,\partial_{\mu}\Phi\,\partial^{\mu}\Phi
\tag{35}
$$

This dynamics represents a massless spinor field coupled non-minimally with gravity. The cosmological constant represents the influence of the rest-of-the-universe on Ψ.

Independent variation of Ψ and $g_{\mu\nu}$ yields

$$
i\gamma^{\mu}\nabla_{\mu}\Psi + (R\,V' - H'\,\partial_{\mu}\Phi\,\partial^{\mu}\Phi - 2H\Box\Phi)\,\Psi = 0
\tag{36}
$$

$$
\alpha_0\left(R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu}\right) = -T_{\mu\nu}
\tag{37}
$$

where $V' \equiv \partial V/\partial \Phi$. The energy-momentum tensor is given by

$$
\begin{aligned}
T_{\mu\nu} \;=\; & \frac{i}{4}\,\bar{\Psi}\gamma_{(\mu}\nabla_{\nu)}\Psi - \frac{i}{4}\nabla_{(\mu}\bar{\Psi}\gamma_{\nu)}\Psi \\
& + \; 2V\left(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}\right) + 2\nabla_\mu\nabla_\nu V - 2\Box V\,g_{\mu\nu} \\
& + \; 2H\,\partial_\mu\Phi\,\partial_\nu\Phi - H\,\partial_\lambda\Phi\,\partial^\lambda\Phi\,g_{\mu\nu} + \frac{\alpha_0}{2}\,\Lambda\,g_{\mu\nu}
\end{aligned}
\tag{38}
$$

Taking the trace of equation (37) we obtain after some algebraic manipulation:

$$
\begin{aligned}
(\alpha_0 + 2V + V')\,R \;=\; & H'\,\Phi\,\partial_\alpha\Phi\,\partial^\alpha\Phi \\
& + \; 2H\,\Phi\,\Box\Phi - 6\Box V + 2\,\alpha_0\,\Lambda
\end{aligned}
\tag{39}
$$

Inserting this result back on the equation (36) yields

$$
i\gamma^\mu\nabla_\mu\,\Psi + \left(\mathbb{X}\,\partial_\lambda\Phi\,\partial^\lambda\Phi + \mathbb{Y}\,\Box\Phi\right)\,\Psi + \mathbb{Z}\,\Psi = 0
\tag{40}
$$

where

$$
\mathbb{Z} \equiv \frac{2\,\alpha_0\,\Lambda\,V'}{Q}
$$

$$
\mathbb{X} = \frac{V'\left(\Phi\,H' - 2H - 6V''\right)}{Q} - H'
$$

$$
\mathbb{Y} = \frac{V'\left(2H\,\Phi - 6V'\right)}{Q} - 2H
$$

where $Q \equiv \alpha_0 + 2V + \Phi\,V'$.

At this stage it is worth selecting among all possible candidates of V and H particular ones that makes the factor on the gradient and on $\Box$ of the field to disappear from equation (40). The simplest way is to set $\mathbb{X} = \mathbb{Y} = 0$ which imply only one condition, that is

$$
H = \frac{-\,3(V')^2}{\alpha_0 + 2V}
\tag{41}
$$

The non-minimal term V is such that $\mathbb{Z}$ reduces to a constant, that is

$$
V = \frac{\alpha_0}{2}\,\left[(1 + \sigma\,\Phi)^{-2} - 1\right]
\tag{42}
$$

Then it follows immediately that

$$
H = -3\alpha_0\,\sigma^2\,(1 + \sigma\,\Phi)^{-4}
\tag{43}
$$

where σ is a constant.

The equation for the spinor becomes

$$i\gamma^\mu \nabla_\mu \Psi - m\Psi = 0 \tag{44}$$

where

$$m = \frac{4\,\sigma\,\Lambda}{\kappa\,c^2}. \tag{45}$$

Thus as a result of the above process the spinor field acquires a mass m that depends crucially on the existence of Λ. If Λ vanishes then the mass of the field vanishes. The non-minimal coupling of gravity with the spinor field corresponds to a specific self-interaction. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe — represented by the cosmological constant. The values of different masses for different fields are contemplated in the parameter σ.

The various steps of our mechanism can be synthesized as follows:

- The dynamics of a massles spinor field Ψ is described by the Lagrangian

$$L_D = \frac{i}{2}\bar{\Psi}\gamma^\mu \nabla_\mu \Psi - \frac{i}{2}\nabla_\mu \bar{\Psi}\gamma^\mu \Psi;$$

- Gravity is described in General Relativity by the scalar of curvature

$$L_E = R;$$

- The field interacts with gravity in a non-minimal way described by the term

$$L_{int} = V(\Phi)\,R$$

where $\Phi = \bar{\Psi}\,\Psi$;

- The action of the rest-of-the-universe on the spinor field, through the gravitational intermediary, is contained in the form of an additional constant term on the Lagrangian noted as Λ;

- A counter-term depending on the invariant Φ is introduced to kill extra terms coming from gravitational interaction;

- As a result of this process, after specifying V and H the field acquires a mass being described as

$$i\gamma^\mu \nabla_\mu \Psi - m\Psi = 0$$

where m is given by equation (45) and is zero only if the cosmological constant vanishes.

This procedure allows us to state that the mechanism proposed here is to be understood as a realization of Mach principle according to which the inertia of a body depends on the background of the rest-of-the-universe. This strategy can be applied in a more general context in support of the idea that (local) properties of microphysics may depend on the (global) properties of the universe. We will analyze this in the next session (see also [6]).

Thus, collecting all these terms we obtain the final form of the Lagrangian

$$
\begin{aligned}
L = {} & \frac{i}{2}\bar\Psi\gamma^\mu\nabla_\mu\Psi - \frac{i}{2}\nabla_\mu\bar\Psi\gamma^\mu\Psi \\
& + \frac{1}{\kappa}\,(1+\sigma\,\Phi)^{-2}\,R - \frac{1}{\kappa}\,\Lambda \\
& - \frac{6}{\kappa}\,\sigma^2\,(1+\sigma\,\Phi)^{-4}\,\partial_\mu\Phi\,\partial^\mu\Phi
\end{aligned}
\tag{46}
$$

Some comments

- In the case $\sigma = 0$ the Lagrangian reduces to a massless fermion satisfying Dirac's dynamics plus the gravitational field described by General Relativity;

- The dimensionality of σ is L^3;

- The ratio $m/\sigma = 4\,\Lambda/\kappa\,c^2$ which has the meaning of a density of mass is an universal constant. How to interpret such universality?

A possible solution of this question can be found in the following way. The present mechanism is a real possibility if the mass obtained through it does not depends neither in the intensity of the gravitational field nor on the Newton's constant, as it occurs in the previous case of the scalar field and for the vector field as it will be shown in the next section. In the case of fermions, we have found that the constant of interaction has dimensionality $lenght^3$. Thus one should write $\sigma = \sigma_0\,L_{Planck}^2\,\Lambda^{-1/2}$ where σ_0 is a dimensionless constant. obtaining in this case, for the mass of the fermions, the similar result $\mu = \sigma_0\,\Lambda$.

V. THE CASE OF VECTOR FIELDS

We start with a scenario in which there are only three ingredients: a massless vector field, the gravitational field and an homogeneous distribution of energy - that is identified with the vacuum. The theory is specified by the Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{\kappa} R - \frac{\Lambda}{\kappa} \tag{47}$$

The corresponding equations of motion are

$$F^{\mu\nu}{}_{;\nu} = 0$$

and

$$\alpha_0 \left(R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \right) = -T_{\mu\nu}$$

where $F_{\mu\nu} = \nabla_\nu W_\mu - \nabla_\mu W_\nu$ and $\alpha_0 \equiv 2/\kappa$.

In this theory, the vacuum Λ is invisible for W_μ. The energy distribution represented by Λ interacts with the vector field only indirectly once it modifies the geometry of space-time. In the Higgs mechanism this vacuum is associated to a fundamental state of a scalar field φ and it is transformed in a mass term for W_μ. The role of Λ is displayed by the value of the potential $V(\varphi)$ in its homogeneous state. We will now show that there is no needs to introduce any extra scalar field by using the universal character of gravitational interaction to generate mass for W_μ.

The point of departure is the recognition that gravity may be the real responsible for breaking the gauge symmetry. For this, we modify the above Lagrangian to include a non-minimal coupling of the field W_μ to gravity in order to explicitly break such invariance. There are only two possible ways for this [13]. The total Lagrangian must be of the form

$$\begin{aligned}
\mathbb{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{\kappa} R \\
& + \frac{\gamma}{6} R \, \Phi + \gamma \, R_{\mu\nu} W^\mu W^\nu \\
& - \frac{\Lambda}{\kappa}
\end{aligned} \tag{48}$$

where we define

$$\Phi \equiv W_\mu W^\mu.$$

The first two terms of $\mathbb{L}$ represents the free part of the vector and the gravitational fields. The second line represents the non-minimal coupling interaction of the vector field with gravity. The parameter σ is dimensionless. The vacuum – represented by Λ – is added by the reasons presented above and it must be understood as the definition of the expression "the influence of the rest-of-the-universe on W_μ". We will not make any further hypothesis on this [14].

In the present proposed mechanism, Λ is the real responsible to provide mass for the vector field. This means that if we set $\Lambda = 0$ the mass of W_μ will vanish.

Independent variation of W_μ and $g_{\mu\nu}$ yields

$$F^{\mu\nu}{}_{;\nu} + \frac{\gamma}{3} R W^\mu + 2\gamma R^{\mu\nu} W_\nu = 0 \tag{49}$$

$$\alpha_0 \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) = -T_{\mu\nu} \tag{50}$$

The energy-momentum tensor defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta g^{\mu\nu}}$$

is given by

$$
\begin{aligned}
T_{\mu\nu} =\ & E_{\mu\nu} \\
& + \frac{\gamma}{3} \nabla_\mu \nabla_\nu \Phi - \frac{\gamma}{3} \Box\Phi\, g_{\mu\nu} + \frac{\gamma}{3} \Phi \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) \\
& + \frac{\gamma}{3} R W_\mu W_\nu + 2\gamma R_\mu{}^\lambda W_\lambda W_\nu + 2\gamma R_\nu{}^\lambda W_\lambda W_\mu \\
& - \gamma R_{\alpha\beta} W^\alpha W^\beta g_{\mu\nu} - \gamma \nabla_\alpha \nabla_\beta (W^\alpha W^\beta) g_{\mu\nu} \\
& + \gamma \nabla_\nu \nabla_\beta (W_\mu W^\beta) + \gamma \nabla_\mu \nabla_\beta (W_\nu W^\beta) \\
& + \gamma \Box(W_\mu W_\nu) + \frac{1}{\kappa} \Lambda\, g_{\mu\nu}
\end{aligned} \tag{51}
$$

where

$$E_{\mu\nu} = F_{\mu\alpha} F^\alpha{}_\nu + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu}$$

Taking the trace of equation (50) we obtain

$$R = 2\Lambda - \kappa\gamma \nabla_\alpha \nabla_\beta (W^\alpha W^\beta) \tag{52}$$

Then, using this result back into equation (49) it follows

$$F^{\mu\nu}{}_{;\nu} + \frac{2\,\gamma\,\Lambda}{3}\,W^{\mu}$$
$$- \frac{\kappa\,\gamma^2}{3}\,\nabla_{\alpha}\,\nabla_{\beta}\,(W^{\alpha}\,W^{\beta})\,W^{\mu}$$
$$+ 2\,\gamma\,R^{\mu}{}_{\nu}\,W^{\nu} = 0 \tag{53}$$

The non-minimal coupling with gravity yields an effective self-interaction of the vector field and a term that represents its direct interaction with the curvature of space-time. Besides, as a result of this process the vector field acquires a mass μ that depends on the constant γ and on the existence of λ. The term

$$2\,\gamma\,R^{\mu}{}_{\nu}\,W^{\nu}$$

gives a contribution (through the dynamics of the metric equation (50) of $\gamma\,\Lambda$ yielding for the mass the formula

$$\mu^2 = \frac{5}{3}\,\gamma\,\Lambda \tag{54}$$

Note that the Newton's constant does not appear in our formula for the mass. The net effect of the non-minimal coupling of gravity with W^{μ} corresponds to a specific self-interaction of the vector field. The mass of the field appears only if we take into account the existence of the rest-of-the-universe — represented by Λ — in the state in which this environment is on the corresponding vacuum. If Λ vanishes then the mass of the field vanishes. The values of different masses for different fields are contemplated in the parameter γ.

Quantum perturbations

How this process that we have been examining here to give mass to all kind of bodies should be modified in a quantum version? We note, first of all, that the gravitational field is to be treated at a classical level, once there is neither theoretical nor observational evidence that exists a quantum version of gravitational interaction. Thus, any modification of the present scheme means to introduce quantum aspects of the vector field. This will not change the whole scheme of generation of mass described above. Indeed, in the semi-classical approach in which the matter field is quantized but the metric is not, the modification of the equation of general relativity becomes

$$\alpha_0\,(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) = -\,<T_{\mu\nu}> \tag{55}$$

where the field is in a given specific state. Throughout all the process of gravitational interaction the system does not change its state, allowing the same classical treatment as above.

VI.   THE CASE OF SPIN-TWO FIELD

As in the previous cases we start with a scenario in which there are only three ingredients: a linear tensor field, the gravitational field and an homogeneous distribution of energy identified with the vacuum. We note that there are two possible equivalent ways to describe a spin-two field that is:

- Einstein frame

- Fierz frame

according we use a symmetric second order tensor $\varphi_{\mu\nu}$ or the third-order tensor tensor $F_{\alpha\beta\lambda}$. Although the Fierz representation is not used for most of the works dealing with spin-2 field, it is far better than the Einstein frame when dealing in a curved space-time[16]. Thus, let us review briefly the basic properties of the Fierz frame[22]. We start by defining a three-index tensor $F_{\alpha\beta\mu}$ which is anti-symmetric in the first pair of indices and obeys the cyclic identity:

$$F_{\alpha\mu\nu} + F_{\mu\alpha\nu} = 0, \tag{56}$$

$$F_{\alpha\mu\nu} + F_{\mu\nu\alpha} + F_{\nu\alpha\mu} = 0. \tag{57}$$

This expression implies that the dual of $F_{\alpha\mu\nu}$ is trace-free:

$$\overset{*}{F}{}^{\alpha\mu}{}_{\mu} = 0, \tag{58}$$

where the asterisk represents the dual operator, defined in terms of $\eta_{\alpha\beta\mu\nu}$ by

$$\overset{*}{F}{}^{\alpha\mu}{}_{\lambda} \equiv \frac{1}{2}\,\eta^{\alpha\mu}{}_{\nu\sigma}\,F^{\nu\sigma}{}_{\lambda}.$$

The tensor $F_{\alpha\mu\nu}$ has 20 independent components. The necessary and sufficient condition for $F_{\alpha\mu\nu}$ to represent an unique spin-2 field (described by 10 components) is [23]

$$\overset{*}{F}{}^{\alpha(\mu\nu)}{}_{,\alpha} = 0, \tag{59}$$

which can be rewritten as

$$F_{\alpha\beta}{}^{\lambda}{}_{,\mu} + F_{\beta\mu}{}^{\lambda}{}_{,\alpha} + F_{\mu\alpha}{}^{\lambda}{}_{,\beta} - \frac{1}{2}\delta_{\alpha}^{\lambda}(F_{\mu,\beta} - F_{\beta,\mu}) +$$

$$-\frac{1}{2}\delta_{\mu}^{\lambda}(F_{\beta,\alpha} - F_{\alpha,\beta}) - \frac{1}{2}\delta_{\beta}^{\lambda}(F_{\alpha,\mu} - F_{\mu,\alpha}) = 0. \tag{60}$$

A direct consequence of the above equation is the identity:

$$F^{\alpha\beta\mu}{}_{,\mu} = 0 \ . \tag{61}$$

We call a tensor that satisfies the conditions given in the Eqns.(56), (57) and (59) a Fierz tensor. If $F_{\alpha\mu\nu}$ is a Fierz tensor, it represents an unique spin-2 field. Condition (59) yields a connection between the Einstein frame (EF) and the Fierz frame (FF): it implies that there exists a symmetric second-order tensor $\varphi_{\mu\nu}$ that acts as a potential for the field. We write

$$\begin{aligned} 2\, F_{\alpha\mu\nu} &= \varphi_{\nu[\alpha,\mu]} + \left(\varphi_{,\alpha} - \varphi_{\alpha}{}^{\lambda}{}_{,\lambda}\right)\eta_{\mu\nu} \\ &\quad - \left(\varphi_{,\mu} - \varphi_{\mu}{}^{\lambda}{}_{,\lambda}\right)\eta_{\alpha\nu}. \end{aligned} \tag{62}$$

where $\eta_{\mu\nu}$ is the flat spacetime metric tensor, and the factor 2 in the l.h.s. is introduced for convenience.

Taking the trace of equation (62) $F_{\alpha} \equiv F_{\alpha\mu\nu}\eta^{\mu\nu}$ it follows that

$$F_{\alpha} = \varphi_{,\alpha} - \varphi_{\alpha}{}^{\lambda}{}_{,\lambda},$$

where . Thus we can write

$$2F_{\alpha\mu\nu} = \varphi_{\nu[\alpha,\mu]} + F_{[\alpha}\,\eta_{\mu]\nu}. \tag{63}$$

Using the properties of the Fierz tensor we obtain the important identity:

$$F^{\alpha}{}_{(\mu\nu),\alpha} \equiv -\,2\,G^{(L)}{}_{\mu\nu}, \tag{64}$$

where $G^{(L)}{}_{\mu\nu}$ is the linearized Einstein tensor, defined by the perturbation $g_{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}$ by

$$2\,G^{(L)}{}_{\mu\nu} \equiv \Box\,\varphi_{\mu\nu} - \varphi^{\epsilon}{}_{(\mu,\nu)\,,\epsilon} + \varphi_{,\mu\nu} - \eta_{\mu\nu}\left(\Box\varphi - \varphi^{\alpha\beta}{}_{,\alpha\beta}\right). \tag{65}$$

The divergence of $F^{\alpha}{}_{(\mu\nu),\alpha}$ yields Bianci identity:

$$F^{\alpha(\mu\nu)}{}_{,\alpha\mu} \equiv 0. \tag{66}$$

Indeed,

$$F^{\alpha\mu\nu}{}_{,\alpha\mu} + F^{\alpha\nu\mu}{}_{,\mu\alpha} = 0. \tag{67}$$

The first term vanishes identically due to the symmetric properties of the field and the second term vanishes due to equation (61). Using Eqn.(64) the identity which states that the linearized Einstein tensor $G^{(L)}{}_{\mu\nu}$ is divergence-free is recovered.

We shall build now dynamical equations for the free Fierz tensor in flat spacetime. Our considerations will be restricted here to linear dynamics. The most general theory can be constructed from a combination of the three invariants involving the field. These are represented by A, B and W:

$$A \equiv F_{\alpha\mu\nu} F^{\alpha\mu\nu}, \qquad B \equiv F_\mu F^\mu,$$

$$W \equiv F_{\alpha\beta\lambda} \overset{*}{F}{}^{\alpha\beta\lambda} = \frac{1}{2} F_{\alpha\beta\lambda} F^{\mu\nu\lambda} \eta^{\alpha\beta}{}_{\mu\nu}.$$

W is a topological invariant so we shall use only the invariants A and B. The EOM for the massless spin-2 field in the ER is given by

$$G^{(L)}{}_{\mu\nu} = 0. \tag{68}$$

As we have seen above, in terms of the field $F^{\lambda\mu\nu}$ this equation can be written as

$$F^{\lambda(\mu\nu)}{}_{,\lambda} = 0. \tag{69}$$

The corresponding action takes the form

$$S = \frac{1}{k} \int \mathrm{d}^4 x \, (A - B). \tag{70}$$

Then,

$$\delta S = \int F^{\alpha\,(\mu\nu)}{}_{,\alpha} \, \delta\varphi_{\mu\nu} \, d^4 x. \tag{71}$$

we obtain

$$\delta S = -2 \int G^{(L)}{}_{\mu\nu} \, \delta\varphi^{\mu\nu} \, d^4 x, \tag{72}$$

where $G^{(L)}{}_{\mu\nu}$ is given in Eqn.(65).

Let us consider now the massive case. If we include a mass for the spin 2 field in the Fierz frame, the Lagrangian takes the form

$$\mathcal{L} = A - B + \frac{m^2}{2} \left(\varphi_{\mu\nu} \, \varphi^{\mu\nu} - \varphi^2 \right), \tag{73}$$

and the EOM that follow are

$$F^{\alpha}{}_{(\mu\nu),\alpha} - m^2 \left(\varphi_{\mu\nu} - \varphi\,\eta_{\mu\nu}\right) = 0, \tag{74}$$

or equivalently,

$$G^{(L)}{}_{\mu\nu} + \frac{m^2}{2}\left(\varphi_{\mu\nu} - \varphi\,\eta_{\mu\nu}\right) = 0.$$

The trace of this equation gives

$$F^{\alpha}{}_{,\alpha} + \frac{3}{2}\,m^2\,\varphi = 0, \tag{75}$$

while the divergence of Eqn.(74) yields

$$F_\mu = 0. \tag{76}$$

This result together with the trace equation gives $\varphi = 0$.

In terms of the potential, Eqn.(76) is equivalent to

$$\varphi_{,\mu} - \varphi^{\epsilon}{}_{\mu,\epsilon} = 0. \tag{77}$$

It follows that we must have

$$\varphi^{\mu\nu}{}_{,\nu} = 0.$$

Thus we have shown that the original ten degrees of freedom (DOF) of $F_{\alpha\beta\mu}$ have been reduced to five (which is the correct number for a massive spin-2 field) by means of the five constraints

$$\varphi^{\mu\nu}{}_{,\nu} = 0, \qquad \varphi = 0. \tag{78}$$

Equation of spin-2 in curved background

The passage of the spin-2 field equation from Minkowski spacetime to arbitrary curved riemannian manifold presents ambiguities due to the presence of second order derivatives of the rank two symmetric tensor $\varphi_{\mu\nu}$ that is used in the so called Einstein-frame (see for instance [?]). These ambiguities disappear when we pass to the Fierz frame representation that deals with the three index tensor $F_{\alpha\mu\nu}$ as it was shown in [?].

There results a unique form of minimal coupling, free of ambiguities. Let us define from $\varphi_{\mu\nu}$ two auxiliary fields $G^{(I)}{}_{\mu\nu}$ and $G^{(II)}{}_{\mu\nu}$ through the expressions:

$$2\,G^{(I)}{}_{\mu\nu} \equiv$$
$$\Box\,\varphi_{\mu\nu} - \varphi_{\epsilon(\mu;\nu)}{}^{;\epsilon} + \varphi_{;\mu\nu} - \eta_{\mu\nu}\left(\Box\varphi - \varphi^{\alpha\beta}{}_{;\alpha\beta}\right), \tag{79}$$
$$2\,G^{(II)}{}_{\mu\nu} \equiv$$
$$\Box\,\varphi_{\mu\nu} - \varphi_{\epsilon(\mu}{}^{;\epsilon}{}_{;\nu)} + \varphi_{;\mu\nu} - \eta_{\mu\nu}\left(\Box\varphi - \varphi^{\alpha\beta}{}_{;\alpha\beta}\right). \tag{80}$$

These objects differ only in the order of the second derivative in the second term on the r.h.s. of the above equations. The equation of motion [?] free of ambiguities concerns the tensor field

$$\widehat{G}_{\mu\nu} \equiv \frac{1}{2}\left(G^{(I)}{}_{\mu\nu} + G^{(II)}{}_{\mu\nu}\right) \tag{81}$$

and is given by

$$\widehat{G}_{\mu\nu} + \frac{1}{2}\,m^2\left(\varphi_{\mu\nu} - \varphi g_{\mu\nu}\right) = 0. \tag{82}$$

which is precisely the usual equations for massive spin-2 field.

Generating mass for the spin-2 field

We follow the same strategy as in the previous case and take the dynamics of the spin-2 field as given by

$$\begin{aligned}
\mathbb{L} &= F_{\alpha\mu\nu}\,F^{\alpha\mu\nu} - F_\alpha\,F^\alpha + \frac{1}{\kappa}\,R \\
&+ a\,R_{\alpha\mu\beta\nu}\,\varphi^{\alpha\beta}\,\varphi^{\mu\nu} \\
&- \frac{\Lambda}{\kappa}
\end{aligned} \tag{83}$$

The equations of motion are given by:

$$F^\alpha{}_{(\mu\nu);\alpha} + 2a\,R_{\alpha\mu\beta\nu}\,\varphi^{\alpha\beta} = 0, \tag{84}$$

$$\frac{1}{\kappa}\left(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} + \frac{\Lambda}{2}\,g_{\mu\nu}\right) + T_{\mu\nu} + a\,Y_{\mu\nu} = 0 \tag{85}$$

where the quantity $Y_{\mu\nu}$ is given by the variation of the non minimal coupling term:

$$\delta \int \sqrt{-g}\, R_{\alpha\mu\beta\nu}\, \varphi^{\alpha\beta}\, \varphi^{\mu\nu} = \int \sqrt{-g}\, Y_{\mu\nu}\, \delta g_{\mu\nu} \tag{86}$$

where $Y_{\mu\nu}$ is given in terms of $S_{\alpha\mu\beta\nu}$ defined as

$$S_{\alpha\mu\beta\nu} \equiv \varphi_{\alpha\beta}\, \varphi_{\mu\nu} - \varphi_{\alpha\nu}\, \varphi_{\beta\mu}$$

which has the symmetries:

$$S_{\alpha\mu\beta\nu} = -\, S_{\alpha\mu\nu\beta} = -\, S_{\mu\alpha\beta\nu} = S_{\beta\nu\alpha\mu}.$$

A direct calculation yields

$$Y^{\mu\nu} \equiv S^{\lambda\mu\nu\epsilon}{}_{;\epsilon;\lambda} - \frac{1}{2}\, R_{\alpha\sigma\beta\lambda}\, \varphi^{\alpha\beta}\, \varphi^{\sigma\lambda}\, g^{\mu\nu} + \frac{3}{2}\, R_{\alpha\sigma\beta}{}^{(\mu}\, \varphi^{\nu)\sigma}\, \varphi^{\alpha\beta} - \frac{1}{2}\, R_{\alpha\sigma\beta\lambda}\, \varphi^{\alpha\beta}\, \varphi^{\sigma\lambda}\, g_{\mu\nu}$$

Let us remind that the Riemann curvature can be written in terms of its irreducible quantities involving the Weyl conformal tensor $W_{\alpha\sigma\beta\lambda}$ and the contracted Ricci tensor by the formula:

$$R_{\alpha\mu\beta\nu} = W_{\alpha\mu\beta\nu} + \frac{1}{2}\left(R_{\alpha\beta}\, g_{\mu\nu} + R_{\mu\nu}\, g_{\alpha\beta} - R_{\alpha\nu}\, g_{\beta\mu} - R_{\beta\mu}\, g_{\alpha\nu}\right) - \frac{1}{6}\, R\, g_{\alpha\mu\beta\nu}.$$

Then

$$R_{\alpha\mu\beta\nu}\, \varphi^{\alpha\beta}\, \varphi^{\mu\nu} = W_{\alpha\mu\beta\nu}\, \varphi^{\alpha\beta}\, \varphi^{\mu\nu} + \left(R_{\alpha\beta} - \frac{1}{6}\, R\, g_{\alpha\beta}\right)\left(\varphi\, \varphi^{\alpha\beta} - \varphi^{\alpha}{}_{\lambda}\, \varphi^{\lambda\beta}\right).$$

We can then re-write the equation of the spin-2 field as

$$F^{\alpha}{}_{(\mu\nu);\alpha} - \frac{a\,\Lambda}{3}\left(\varphi_{\mu\nu} - \varphi\, g_{\mu\nu}\right) + 2a\, W_{\alpha\mu\beta\nu}\, \varphi^{\alpha\beta} + Q_{\mu\nu} = 0, \tag{87}$$

where $Q_{\mu\nu}$ contain non-linear terms of interaction of the spin-2 field with gravity.

VII.   GENERALIZED MACH'S PRINCIPLE

In this section we present an extension of Mach principle in similar lines as it has been suggested by Dirac, Hoyle and others. This generalization aims to produce a mechanism that transforms the vague idea according to which local properties may depend on the universe's global characteristics into an efficient process. We will apply the strategy that we used in the precedent sections to generate mass in order to elaborate such generalization.

The cosmological influence on the microphysical world: the case of chiral-invariant Heisenberg-Nambu-Jona-Lasinio dynamics

There have been many discussions in the scientific literature in the last decades related to the cosmic dependence of the fundamental interactions. The most popular one was the suggestion of Dirac – the so called Large Number Hypothesis – that was converted by Dicke and Brans into a new theory of gravitation, named the scalar-tensor theory. We will do not analyze any of these here. On the contrary, we will concentrate on a specific self-interaction of an elementary field and show that its correspondent dynamics is a consequence of a dynamical cosmological process. That is, to show that dynamics of elementary fields in the realm of microphysics, may depend on the global structure of the universe.

The first question we have to face concerns the choice of the elementary process. There is no better way than start our analysis with the fundamental theory proposed by Nambu and Jona-Lasinio concerning a dynamical model of elementary particles [19]. Since the original paper until to-day hundreds of papers devoted to the NJL model were published [18]. For our purpose here it is enough to analyze the nonlinear equation of motion that they used in their original paper as the basis of their theory which is given by

$$i\gamma^\mu \nabla_\mu \Psi - 2s(A + i\,B\,\gamma^5)\Psi = 0$$

This equation, as remarked by these authors, was proposed earlier by Heisenberg [20] although in a quite different context. We will not enter in the analysis of the theory that follows from this dynamics. Our question here is just this: is it possible to produce a model such that HNJL (Heisenberg-Nambu-Jona-Lasinio) equation for spinor field becomes a consequence of the gravitational interaction of a free massless Dirac field with the rest-of-the-universe? We shall see that the answer is yes.

We used Mach's principle as the statement according to which the inertial properties of a body $\mathbb{A}$ are determined by the energy-momentum throughout all space. We follow here a similar procedure and will understand the Extended Mach Principle as the idea which states that the influence of the rest-of-the-universe on microphysics can be described through the action of the energy-momentum distribution identified with the cosmic form

$$T^U_{\mu\nu} = \Lambda\,g_{\mu\nu}$$

Non minimal coupling with gravity

In the framework of General Relativity we set the dynamics of a fermion field Ψ coupled non-minimally with gravity to be given by the Lagrangian (we are using units were $\hbar = c = 1$)

$$L = L_D + \frac{1}{\kappa} R + V(X) R - \frac{1}{\kappa} \Lambda + L_{CT} \tag{88}$$

where

$$L_D \equiv \frac{i}{2} \bar{\Psi} \gamma^\mu \nabla_\mu \Psi - \frac{i}{2} \nabla_\mu \bar{\Psi} \gamma^\mu \Psi \tag{89}$$

The non-minimal coupling of the spinor field with gravity is contained in the term $V(X)$ and depends on the scalar X defined by

$$X = A^2 + B^2$$

where $A = \bar{\Psi} \Psi$ and $B = i \bar{\Psi} \gamma^5 \Psi$. We note that we can write, in an equivalent way,

$$X = J_\mu J^\mu$$

where $J^\mu = \bar{\Psi} \gamma^\mu \Psi$. This quantity X is chiral invariant, once it is invariant under the map

$$\Psi' = \gamma^5 \Psi.$$

Indeed, from this γ^5 transformation, it follows

$$A' = -A, \ B' = -B; \ then, X' = X.$$

The case in which the theory breaks chiral invariance and the interacting term V depends only on the invariant A – is the road to the appearance of a mass as we saw in the previous sections [6]. Here we start from the beginning with a chiral invariant theory. For the time being the dependence of V on X is not fixed. We have added L_{CT} to counter-balance the terms of the form $\partial_\lambda X \, \partial^\lambda X$ and $\Box X$ that appear due to the gravitational interaction. The most general form of this counter-term is

$$L_{CT} = H(X) \, \partial_\mu X \, \partial^\mu X \tag{90}$$

We shall see that H depends on V and if we set $V = 0$ then H vanishes. This dynamics represents a massless spinor field coupled non-minimally with gravity. The cosmological constant represents the influence of the rest-of-the-universe on Ψ.

Independent variation of Ψ and $g_{\mu\nu}$ yields

$$i\gamma^\mu \nabla_\mu \Psi + \Omega\,(A + i\,B\,\gamma^5)\Psi = 0 \tag{91}$$

where

$$\Omega \equiv 2R\,V' - 2H'\,\partial_\mu X\,\partial^\mu X - 4H\Box X$$

$$\alpha_0\,(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) = -T_{\mu\nu} \tag{92}$$

where we set $\alpha_0 \equiv 2/\kappa$ and $V' \equiv \partial V/\partial X$. The energy-momentum tensor is given by

$$\begin{aligned}
T_{\mu\nu} &= \frac{i}{4}\,\bar\Psi\gamma_{(\mu}\nabla_{\nu)}\Psi - \frac{i}{4}\nabla_{(\mu}\bar\Psi\gamma_{\mu)}\Psi \\
&+ 2V(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) + 2\nabla_\mu\nabla_\nu V - 2\Box V g_{\mu\nu} \\
&+ 2H\,\partial_\mu X\,\partial_\nu X - H\,\partial_\lambda X\,\partial^\lambda X\,g_{\mu\nu} + \frac{\alpha_0}{2}\,\Lambda\,g_{\mu\nu}
\end{aligned} \tag{93}$$

Taking the trace of equation (92), after some simplification and using

$$\Box V = V'\,\Box X + V''\,\partial_\mu X\,\partial^\mu X \tag{94}$$

it follows

$$\begin{aligned}
(\alpha_0 + 2V + 2V'\,X)\,R &= (4HX - 6V')\Box X \\
&+ (2H'\,X - 6V'' - 2H)\,\partial_\alpha X\,\partial^\alpha X \\
&+ 2\,\alpha_0\,\Lambda
\end{aligned} \tag{95}$$

Then

$$\begin{aligned}
\Omega &= (\mathbb{M}\,\Box X + \mathbb{N}\,\partial_\mu X\,\partial^\mu X) \\
&+ \frac{4\,\alpha_0\,\Lambda\,V'}{\alpha_0 + 2V + 2V'\,X}
\end{aligned} \tag{96}$$

where

$$\mathbb{M} = \frac{2V'(4HX - 6V')}{\alpha_0 + 2V + 2V'\,X} - 4H$$

$$\mathbb{N} = \frac{2V'\,(2\,X\,H' - 6V'' - 2\,H)}{\alpha_0 + 2V + 2V'\,X} - 2\,H'$$

Defining $\Delta \equiv \alpha_0 + 2V + 2V'X$ we re-write $\mathbb{M}$ and $\mathbb{N}$ as

$$\mathbb{M} = -\frac{4}{\Delta}\left(3V'^2 + H\left(\alpha_0 + 2V\right)\right)$$

$$\mathbb{N} = -\frac{2}{\Delta}\left(3V'^2 + H\left(\alpha_0 + 2V\right)\right)'$$

Inserting this result on the equation (91) yields

$$i\gamma^\mu \nabla_\mu \Psi + \left(\mathbb{M}\,\Box X + \mathbb{N}\,\partial_\lambda X\,\partial^\lambda X\right)\Psi + \mathbb{Z}\,(A + i\,B\,\gamma^5)\Psi = 0 \qquad (97)$$

where

$$\mathbb{Z} = \frac{4\,\alpha_0\,\Lambda\,V'}{\Delta}$$

At this stage it is worth to select among all possible candidates of V and H particular ones that makes the factor on the gradient and on $\Box$ of the field to disappear from equation (97).

The simplest way is to set $\mathbb{M} = \mathbb{N} = 0$, which is satisfied if

$$H = -\frac{3V'^2}{\alpha_0 + 2V}$$

Imposing that $\mathbb{Z}$ must reduce to a constant we obtain

$$V = \frac{1}{\kappa}\left[\frac{1}{1+\beta X} - 1\right]. \qquad (98)$$

As a consequence of this,

$$H = -\frac{3\beta^2}{2\kappa}\frac{1}{(1+\beta X)^3} \qquad (99)$$

where β is a constant. Using equations 97) and 98) the equation for the spinor becomes

$$i\gamma^\mu \nabla_\mu \Psi - 2s(A + i\,B\,\gamma^5)\Psi = 0 \qquad (100)$$

where

$$s = \frac{2\,\beta\,\Lambda}{\kappa(\hbar\,c)}. \qquad (101)$$

Thus as a result of the gravitational interaction the spinor field satisfies Heisenberg-Nambu-Jona-Lasinio equation of motion. This is possible due to the influence of the rest-of-the-Universe on Ψ. If Λ vanishes then the constant of the self-interaction of Ψ vanishes.

The final form of the Lagrangian is provided by

$$L = L_D + \frac{1}{\kappa\,(1+\beta X)}\,R - \frac{1}{\kappa}\,\Lambda - \frac{3\beta^2}{2\kappa}\frac{1}{(1+\beta X)^3}\,\partial_\mu X\,\partial^\mu X \qquad (102)$$

In this section we analyzed the influence of all the material content of the universe on a fermionic field when this content is in two possible states: in one case its energy distribution is zero; in another case it is in a vacuum state represented by the homogeneous distribution $T_{\mu\nu} = \Lambda g_{\mu\nu}$. Note that when Λ vanishes, the dynamics of the field is independent of the global properties of the universe and it reduces to the massless Dirac equation

$$i\gamma^\mu \nabla_\mu \Psi = 0$$

In the second case, the rest-of-the-universe induces on field Ψ the Heisenberg-Nambu-Jona-Lasinio non-linear dynamics

$$i\gamma^\mu \nabla_\mu \Psi - 2s\left(A + iB\gamma^5\right)\Psi = 0.$$

Such scenario shows a mechanism by means of which the rules of the microphysical world depends on the global structure of the universe. It is not hard to envisage others situations in which the above mechanism can be further applied.

VIII.   APPENDIX: VACUUM STATE IN NON-LINEAR THEORIES

Although the cosmological constant was postulated from first principles, quantum field theory gave a simple interpretation of Λ by its association to the fundamental vacuum state. It is possible to describe its origin even classically as a consequence of certain special states of matter. For instance, non linear theories produce classically a vacuum, defined by its distribution of energy-momentum tensor provided by expression (1). Let us review very briefly how this occurs in a specific example. We start by the standard definition of the symmetric energy-momentum tensor as variation of the Lagrangian induced by variation of the metric tensor, that is

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta L \sqrt{-g}}{\delta g^{\mu\nu}} \tag{103}$$

In order to present a specific example, let us concentrate on the case of electromagnetic field in which the Lagrangian depends only on the invariant F defined by

$$F \equiv F_{\mu\nu} F^{\mu\nu}$$

Then, the expression of the energy-momentum tensor is given by

$$T_{\mu\nu} = -4 \, L_F \, F_\mu{}^\alpha \, F_{\alpha\nu} - L \, g_{\mu\nu}. \tag{104}$$

where $L_F = \partial L / \partial F$ represents the derivative of the Lagrangian with respect to the invariant F. The corresponding equation of motion of the field is provided by

$$(L_F \, F^{\mu\nu})_{;\nu} = 0. \tag{105}$$

where the symbol ; represents covariant derivative. This equation admits a particular solution when L_F vanishes for non-null constant value F_0 . When the system is in this state, the corresponding expression of the energy-momentum tensor reduces to

$$T_{\mu\nu} = \Lambda \, g_{\mu\nu}$$

where

$$\Lambda = L_0.$$

The consequences of this state in Cosmology due to non linear theories of Electrodynamics was revisited recently (see [9]). A by-product is the emergence of effective geometries that mimics gravitational processes like, for instance, non-gravitational black holes or analogue expanding universes in laboratory.

Acknowledgements

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[1] See the lectures at CERN of M. Veltman in CERN 97-05 for a historical review.

[2] R. H. Dicke: Mach's principle and a relativistic theory of gravity in Relativity, groups and topology (Les Houches, 1964) Gordon and Breach Publishers.

[3] M. Novello and S E P Bergliaffa in Physics Reports vol 463, n 4, July 2008 and referennces therein.

[4] Francis Halzen and Alan D. Martin *Quarks and Leptons: An introductory course in Modern Particle Physics, John Wiley and Sons, 1964* and references therein.

[5] M.Novello: *A mechanism to generate mass: the case of fermions* (arXiv [astro-ph.CO]: 1003.5126v1).

[6] M.Novello: *A mechanism to generate mass: the case of fermions* (arXiv [astro-ph.CO]: 1003.5126v1).

[7] M.Novello: *A gravitational mechanism which gives mass to the vectorial bosons* (arXiv XXXXX).

[8] Let us remark that it is possible to understand Mach principle in a broad sense. Indeed, for the method of obtaining mass using the gravity mechanism, the notion of Totality (or rest-of-the-universe) admits two alternative interpretations. This is related to the fact that when we deal with the vacuum represented by the distribution of energy by $T_{\mu\nu} = \lambda\, g_{\mu\nu}$ it is completely irrelevant – for the gravitational mechanism of providing mass — if parameter λ has a classical global origin (the Universe) identified with the cosmological constant introduced by Einstein; or a local quantum one (the environment) identified with the vacuum of quantum fields.

[9] M. Novello and E. Goulart in Eletrodinâmica não linear: causalidade e efeitos cosmológicos (2010) Editora Livraria da Fisica, São Paulo (in portuguese).

[10] J. V. Narlikar, *An introduction to Cosmology, Cambridge University Press, 2002.* In this book one can find further references to previous attempts and in particular the beautiful proposal of Hoyle and Narlikar .

[11] At this point one should note that the expression rest-of-the-universe may not have a strict cosmological meaning but instead is a short term to represent the whole background of matter that really affects $\mathbb{A}$.

[12] However this is not mandatory. The term rest-of-the-universe concerns the environment of $\mathbb{A}$, that is the whole domain of influence on $\mathbb{A}$ of the remaining bodies in the universe.

[13] M. Novello and S E P Bergliaffa: *Bouncing Cosmologies*, Physics Report vol 463, n 4 (2008).

[14] There is not any compelling reason to identify this constant with the actual cosmological constant or the value of the critical density $10^{-48} Gev^4$ provided by cosmology.

[15] B. Zumino in: Yang-Mills theory of weak and electromagnetic interaction; CERN-repport

(1971).

[16] M Novello - R P Neves in Class. Quant. Grav. 19:5335-5351, 2002.

[17] V. P. Vizgin and Ya. A. Smorodinskii: *From the equivalence principle to the equation of gravitation* in Soviet Physics Uspekni, vol 22, n 7 (1979) 489.

[18] M. K. Volkov and A. E. Radzhabov: *Forty-fifth anniversary of the Nambu-Jona-Lasinio model* in arXiv:hep-ph/0508263 (2005).

[19] Y. Nambu and G. Jona-Lasinio: *Dynamical model of elementary particles based on an analogy with superconductivity*, Physical review D, vol 122, n 1 (1961) 345.

[20] W. Heisenberg, Z. Naturforsch. 14, 441 (1959) and earlier papers quoted there.

[21] See [**?**] for some exotic scenarios suggested to take the place of Higgs mechanism in case discovery of Higgs boson fails.

[22] We use the notation $A_{(\alpha}B_{\beta)} = A_\alpha B_\beta + A_\beta B_\alpha$, $A_{[\alpha}B_{\beta]} = A_\alpha B_\beta - A_\beta B_\alpha$.

[23] Note that this condition is analogous to that necessary for the existence of a potential A_μ for the EM field, given by $\overset{*}{A}{}^{\alpha\mu}{}_{,\alpha} = 0$.