The two-dimensional Vavilov-Čerenkov radiation

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Abstract

We derive the power spectrum of photons generated by charged particle moving in parallel direction to the graphene-like structure with index of refraction n. Some graphene-like structures, for instance graphene with implanted ions, or, also 2D-glasses, are dielectric media, and it means that it enables the experimental realization of the Vavilov-Čerenkov radiation. We calculate it from the viewpoint of the Schwinger theory of sources.

The fast moving charged particle in a medium when its speed is faster than the speed of light in this medium produces electromagnetic radiation which is called the Vavilov-Čerenkov radiation.

The prediction of Cerenkov radiation came long ago. Heaviside (1889) investigated the possibility of a charged object moving in a medium faster than electromagnetic waves in the same medium becomes a source of directed electromagnetic radiation. Kelvin (1901) presented an idea that the emission of particles is possible at a speed greater than that of light. Somewhat later, Sommerfeld (1904) proposed the hypothetical radiation with a sharp angular distribution. However, in fact, from experimental point of view, the electromagnetic Čerenkov radiation was first observed in the early 1900's by experiments developed by Marie and Pierre Curie when studying radioactivity emission. In essence they observed the emission of a bluish-white light from transparent substances in the neighborhood of strong radioactive source. But the first attempt to understand the origin of this was made by Mallet (1926, 1929a, 1929b) who observed that the light emitted by a variety of transparent bodies placed close to a radioactive source always had the same bluish-white quality, and that the spectrum was continuous, with no line or band structure characteristic of fluorescence.

Unfortunately, these investigations were forgotten for many years. Čerenkov experiments (Čerenkov, 1934) was performed at the suggestion of Vavilov who opened a door to the true physical nature of the this effect[1] (Bolotovskii, 2009).

This radiation was first theoretically interpreted by Tamm and Frank (1937) in the framework of the classical electrodynamics. The source theoretical description of this effect was given by Schwinger et al. (1976) at the zero temperature regime and the classical spectral formula was generalized to the finite temperature situation and for the massive photons by autor (Pardy, 1989; 2002). The Vavilov-Čerenkov effect was also used by author (Pardy, 1997) to possible measurement of the Lorentz contraction.

Let us start with the three dimensional source theory formulation of the problem. Source theory (Schwinger et al., 1976) is the theoretical construction which uses quantum-mechanical particle language. Initially it was constructed for description of the particle physics situations occurring in the high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by photon or graviton respectively.

The basic formula in the source theory is the vacuum to vacuum amplitude:

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W(S)}, \tag{1}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add.

The electromagnetic field is described by the amplitude (1) with the action

$$W(J) = \frac{1}{2c^2}\int(dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{2}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities, where quantity J_μ is conserved. The symbol $D_{+\mu\nu}(x-x')$, is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$| < 0_+|0_- > |^2 = \exp\{-\frac{2}{\hbar}\operatorname{Im} W\} \stackrel{d}{=} \exp\{-\int dt d\omega \frac{P(\omega,t)}{\hbar\omega}\}, \tag{3}$$

where we have introduced the so called power spectral function $P(\omega, t)$ (Schwinger et al., 1976). In order to extract this spectral function from $\operatorname{Im} W$, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x-x')$.

[1]So, the adequate name of this effect is the Vavilov-Čerenkov effect. In the English literature, however, it is usually called the Čerenkov effect.

The electromagnetic field is described by the four-potentials $A^\mu(\varphi, \mathbf{A})$ and it is generated, including a particular choice of gauge, by the four-current $J^\mu(c\varrho, \mathbf{J})$ according to the differential equation, (Schwinger et al., 1976):

$$(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2})A^\mu = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu)J_\nu \tag{4}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu)D_+(x-x'), \tag{5}$$

where $\eta^\mu \equiv (1, \mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x-x')$ was derived by Schwinger et al. (1976) in the following form:

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{6}$$

Using formulas (2), (3), (5) and (6), we get for the power spectral formula the following expression (Schwinger et al., 1976):

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]\times$$

$$\times\left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\}. \tag{7}$$

Now, we are prepared to apply the last formula to the situations of the two dimensional dielectric medium. We derive here the power spectrum of photons generated by charged particle moving in parallel direction to the graphene-like structure with index of refraction n. While the graphene sheet is conductive, some graphene-like structures, for instance graphene with implanted ions, or, also 2D-glasses, are dielectric media, and it means that it enables the experimental realization of the Vavilov-Čerenkov radiation. Some graphene-like structure can be represented by graphene-based polaritonic crystal sheet (Bludov et al., 2012) which can be used to study the Vavilov-Čerenkov effect. We calculate it from the viewpoint of the Schwinger theory of sources.

The charge and current density of electron moving with the velocity $\mathbf{v}$ and charge e is as it is well known:

$$\varrho = e\delta(\mathbf{x} - \mathbf{v}t) \tag{8}$$

$$\mathbf{J} = e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t) \tag{9}$$

In case of the the two dimensional Vavilov-Čerenkov radiation by source theory formulation, the form of equations (2) and (3) is the same with the difference that $\eta^\mu \equiv (1, \mathbf{0})$ has two space components, or $\eta^\mu \equiv (1, 0, 0)$, and the Green function D_+ as the propagator must be determined by the two dimensional procedure. In other words, the Fourier form of this propagator is with $(dk) = dk^0 d\mathbf{k} = dk^0 dk^1 dk^2 = dk^0 k dk d\theta$

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^3} \frac{1}{\mathbf{k}^2 - n^2(k)^2} e^{ik(x-x')}, \tag{10}$$

or, with $R = |\mathbf{x} - \mathbf{x}'|$

$$D_+(x-x') = \frac{1}{(2\pi)^3} \int_0^{2\pi} d\theta \int_0^{\infty} kdk \int_{-\infty}^{\infty} \frac{d\omega}{c} \frac{e^{ikR\cos\theta - i\omega(t-t')}}{k^2 - \frac{n^2\omega^2}{c^2} - i\varepsilon}. \tag{11}$$

Using $\exp(ikR\cos\theta) = \cos(kR\cos\theta) + i\sin(kR\cos\theta)$ and $(z = kR)$

$$\cos(z\cos\theta) = J_0(z) + 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(z)\cos 2n\theta \tag{12}$$

and

$$\sin(z\cos\theta) = \sum_{n=1}^{\infty}(-1)^n J_{2n-1}(z)\cos(2n-1)\theta, \tag{13}$$

where $J_n(z)$ are the Bessel functions (Kuznetsov, 1962), we get after integration over θ:

$$D_+(x-x') = \frac{1}{(2\pi)^2} \int_0^{\infty} kdk \int_{-\infty}^{\infty} \frac{d\omega}{c} \frac{J_0(kR)}{k^2 - \frac{n^2\omega^2}{c^2} - i\varepsilon} e^{-i\omega(t-t')} \tag{14}$$

and it is pedagogically useful to say that the Bessel function $J_0(z)$ has the following expansion (Kuznetsov, 1962):

$$J_0(z) = \sum_{s=0}^{\infty} \frac{(-1)^s z^{2s}}{s!s!2^{2s}}, \tag{15}$$

which is convergent for all z with regard to the d'Alembert convergence criterion.

The ω-integral in (14) can be performed using the residuum theorem after integration in the complex half ω-plane.

The result of such integration is the propagator D_+ in the following form:

$$D_+(x-x') = \frac{i}{2\pi c} \int_0^{\infty} d\omega J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right) e^{-i\omega|t-t'|}. \tag{16}$$

The initial terms of the expansion of the Bessel function with zero index is as follows:

$$J_0(z) = 1 - \frac{z^2}{2^2} + \frac{z^4}{2^2 4^2} - \frac{z^6}{2^2 4^2 6^2} + \frac{z^8}{2^2 4^2 6^2 8^2} - ... \tag{17}$$

The spectral formula for the two dimensional Vavilov-Čerenkov radiation is the analogue of the formula (7), or,

$$P(\omega,t) = -\frac{\omega}{2\pi}\frac{\mu}{n^2} \int d\mathbf{x}d\mathbf{x}'dt' J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right) \cos[\omega(t-t')] \times$$

$$\times \left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\} \tag{18}$$

where the charge density and current involves only two dimensional velocities and integration is also only two dimensional.

The difference is in the replacing mathematical formulas as follows:

$$\frac{\sin \frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|} \quad \longrightarrow \quad J_0\left(\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|\right) \tag{19}$$

So, After insertion the quantities (8) and (9) into (18), we get:

$$P(\omega,t) = \frac{e^2}{2\pi}\frac{\mu\omega v}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\int dt' J_0\left(\frac{nv\omega}{c}|t-t'|\right)\cos[\omega(t-t')], \quad \beta = v/c, \tag{20}$$

where the t'-integration must be performed. Putting $\tau = t' - t$, we get the final formula:

$$P(\omega,t) = \frac{e^2}{2\pi}\frac{\mu\omega v}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\int_{-\infty}^{\infty} d\tau J_0\left(n\beta\omega\tau\right)\cos(\omega\tau), \quad \beta = v/c. \tag{21}$$

The integral in formula (21) is involved in the tables of integrals (Gradshteyn et al. 1962) on page 745, no. 8. Or,

$$J = \int_0^{\infty} dx J_0\left(ax\right)\cos(bx) = \frac{1}{\sqrt{a^2-b^2}}, \quad 0 < b < a$$

$$J = \infty, a = b; \; J = 0, \quad 0 < a < b \tag{22}$$

In our case we have $a = n\beta\omega$ and $b = \omega$. So, the power spectrum in eq. (21) is as follows with $J_0(-z) = J_0(z)$:

$$P = \frac{e^2}{\pi}\frac{\mu v}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\frac{2}{\sqrt{n^2\beta^2-1}}, \quad n\beta > 1, \; \beta = v/c. \tag{23}$$

and

$$P = 0; \; n\beta < 1, \tag{24}$$

which means that the physical meaning of the quantity P is really the Vavilov-Čerenkov radiation. And it is in our case the two dimensional form of this radiation.

While the formula for the three dimensional (3D) Vavilov-Čerenkov radiation is well known from textbooks and monographs, the two dimensional (2D) form of the Vavilov-Čerenkov radiation was derived here. Let us remember, in conclusion, the fundamental features of the 3D Vavilov-Čerenkov radiation:

1) The radiation arises only for particle velocity greater than the velocity of light in the dielectric medium.
2) It depends only on the charge and not on mass of the moving particles
3) The radiation is produced in the visible interval of the light frequencies and partly in the ultraviolet part of the frequency spectrum. The radiation does not exists for very short waves because from the theory of index of refraction n it follows that $n < 1$ in a such situation.
4) The spectral dependency on the frequency is linear for the 3D homogeneous medium.
5) The radiation generated in the given point of the trajectory spreads on the surface of cone with the vertex in this point and with the axis identical with the direction of motion

of the particle. The vertex angle of the cone is given by the relation $\cos\Theta = c/nv$. There is no cone in the 2D dielectric medium

Let us remark that the energy loss of a particle caused by the Vavilov-Čerenkov radiation are approximately equal to 1% of all energy losses in the condensed matter such as the bremsstrahlung and so on. The fundamental importance of the Vavilov-Čerenkov radiation is in its use for the modern detectors of very speed charged particles in the high energy physics. The detection of the Vavilov-Čerenkov radiation enables to detect not only the existence of the particle, however, also the direction of motion and its velocity and according also its charge. The two-dimensional Vavilov-Čerenkov radiation was still not applied, nevertheless it is promising.

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