

Remote sensing through stratified random media using pupil-plane interferometry

Ervin Goldfain

Welch Allyn Inc., Skaneateles Falls, NY 13153

ABSTRACT

We investigate a novel method for the retrieval of an arbitrary amplitude-object which is illuminated from the far-field and sampled through a stratified random medium of unknown statistics. The setup includes two observation paths, a CCD-based imaging system and a multiaperture interferometer placed in a plane conjugate to the entrance pupil of the imaging system. The interferometric baselines are arranged in closed loops to make the closure phase insensitive to random refractive fluctuations. The method may be beneficial to applications such as surveillance, speckle interferometry and biomedical imaging.

Keywords: remote sensing, imaging through turbulence, pupil-plane interferometry, closure phase, digital filtering.

1. INTRODUCTION

A large variety of techniques related to the retrieval of embedded objects have been developed in recent years. They are applied in areas such as the detection of hidden targets using radar technology²⁰⁻²⁵, imaging through atmospheric turbulence⁷⁻¹⁰ and tissue imaging for biomedical research²⁶⁻²⁹. The overall performance of the retrieval process depends heavily on the ability to filter out both the random and the deterministic noise components present in the image (such as time dependent refractive fluctuations, residual scattering on inhomogeneities and absorption throughout the medium). In most cases, random screens⁴ are introduced in computer simulations to model phase and amplitude distortions. The statistics of the random noise is, however, seldom known in advance. This makes the object retrieval dependent on initial assumptions that may be partially invalid. We discuss in this work a dual detection protocol which does not require prior knowledge of noise statistics. The dual setup incorporates a CCD based imaging path and a multiaperture interferometric path, the latter using the phase-closure technique^{7,9-11} to record the cross-spectral density of the radiated field. The retrieval strategy amounts to a four-step procedure, i.e: 1) locate a potential target, 2) measure the image spectrum from the CCD image, 3) evaluate the cross-spectral density using the interference fringe pattern 4) perform object matching.

The paper is organized as follows: we first model the stratified random medium as an equivalent phase filter and relate its transfer function to the unknown object and noise spectra. We then use the relationship between the cross-spectral density of the transmitted field at large distances from the object and the transfer function of the filter³. Using the phase-closure method, we establish a direct correlation between the fringe pattern and the object irradiance which is independent from the noise spectrum. Results are summarized in the last section.

2. THEORETICAL FRAMEWORK

With reference to fig.1, we consider a planar amplitude-object embedded inside a stratified medium with a randomly fluctuating refractive index distribution $n(z,t)$ where "z" represents the stratification direction and "t" is the time. To simplify the following derivation, we assume that the average absorption properties of the stratified medium are stationary and homogeneous. Let the medium be illuminated from the far-field and viewed through the imaging path depicted in fig. 2. The medium may then be simulated by an equivalent noisy spatial frequency filter^{1,2}, i.e.:

$$h(x,y)_m = h(x,y) + n(x,y)_m \quad (1)$$

where $h(x,y)$ is the undistorted point spread function, $n(x,y)_m$ is the additive noise created by random phase fluctuations and $h(x,y)_m$ the instantaneous point spread function. The CCD image is given by:

$$i(x,y) = h(x,y) * o(x,y) + n(x,y)_d \quad (2)$$

in which $o(x,y)$ represents the unknown object intensity function, $*$ is the convolution operator and $n(x,y)_d$ the generic detection noise introduced by optical aberrations, photon-noise and digitization. Taking the Fourier transform of (2) yields:

$$I(f_x, f_y) = [H(f_x, f_y).O(f_x, f_y) + N(f_x, f_y)_m.O(f_x, f_y)] + N(f_x, f_y)_d \quad (3)$$

where f_x, f_y are spatial frequency variables. In what follows we assume that the object is described as a secondary source of nearly monochromatic and unpolarized light that can be considered stationary at least in the wide sense^{3,4}. The cross-spectral density function of the transmitted field in the half-space $z \gg z_0 + h$, where h represents the thickness of the stratified medium, is given by³:

$$w(r_1, s_1, r_2, s_2, \omega) = (2.\pi/k.r)^2 . s_{1z} . s_{2z} . T^*(s_{1p}, \omega) . T(s_{2p}, \omega) . A(s_{1p}, s_{2p}, \omega) \quad (4)$$

Here k stands for the average vacuum wave number, $s = (s_x, s_y, s_z)$ is the unit vector along a representative raypath, $s_p = (s_x, s_y, 0)$, s_1 and s_2 are direction vectors for two arbitrary field points, $r = (x, y, z)$, r locates the midpoint :

$$r = | (r_1 + r_2) / 2 | \quad (5)$$

ω is the light average frequency, $T^*()$ and $T()$ are complex transmission functions through the medium and $A()$ the angular correlation function of the field incident on the medium ($z = z_0$):

$$A(s_{1p}, s_{2p}, \omega) = \langle a^*(s_{1p}, \omega) a(s_{2p}, \omega) \rangle_\omega \quad (6)$$

in which $a^*()$ and $a()$ stand for the angular spectral amplitudes. The angular correlation function is directly related to the coherence properties of the object via:

$$A(s_{1p}, s_{2p}, \omega) = k^4 . W^{(0)}(f_1, f_2, \omega) \quad (7)$$

where $W^{(0)}()$ is the Fourier transform (FT) of the object cross-spectral density function at $z = 0$, i.e.:

$$W^{(0)}(f_1, f_2, \omega) = FT [W^{(0)}(r_{10}, r_{20}, \omega)] \quad (8)$$

In the above r_{10} and r_{20} locate two arbitrary object points at $z = 0$ and the spatial frequencies f_1, f_2 are given by:

$$f_1 = -k . s_{1p} \quad (9)$$

$$f_2 = k . s_{2p}$$

To further simplify the derivation, we confine ourselves to the case of an incoherent and isotropically emitting object. It follows from (7) and (8) that , for a given ω and up to a multiplicative constant, the angular correlation function of such a source is identical with its Fourier spectrum, i.e.:

$$A^{incoh}(s_p) = (const) . O(f) \quad (10)$$

$$| f_1 | = | f_2 | = f$$

Replacing (10) in (4) we obtain:

$$w(r_1, s_1, r_2, s_2, \omega) = (const) . s_{1z} . s_{2z} . T^*(s_{1p}, \omega) . T(s_{2p}, \omega) . O(f) \quad (11)$$

We now take the statistical average of (11) and note that, since the instantaneous transmission and the angular correlation functions are independent random variables, the average of the product equals the product of the averages. Hence we have:

$$\langle w(r_1, s_1, r_2, s_2, \omega) \rangle = (\text{const})_{s_{1z}, s_{2z}} \langle T^*(s_{1p}, \omega) \cdot T(s_{2p}, \omega) \rangle \cdot \langle O(f) \rangle \quad (12)$$

According to the Wiener-Khinchin theorem, Fourier transforming the average spatial autocorrelation of the transmission function recovers the power spectral density of the phase filter^{4,5}. It follows that, for a given ω , we obtain:

$$\text{FT}(\langle T^*(s_{1p}) \cdot T(s_{2p}) \rangle) = \text{FT}(\langle \exp j \cdot [\varphi(s_{2p}) - \varphi(s_{1p})] \rangle) = (\text{const}) \cdot \langle |N(f)_m|^2 \rangle \quad (13)$$

in which φ represents the random phase shift introduced by the filter statistics. In (13) the noise spectrum is defined by:

$$N(f)_m = \text{FT}[n(s_p)_m] = \text{FT}\{\exp[j \cdot \varphi(s_p)]\} \quad (14)$$

The noise power spectrum $|N(f)_m|^2$ may be computed from (3) as:

$$|N(f)_m|^2 = N(f)_m \cdot N^*(f)_m = [I(f) \cdot O^{-1}(f) - N_d(f) \cdot O^{-1}(f) - H(f)] \cdot [I^*(f) \cdot O^{*-1}(f) - N_d^*(f) \cdot O^{*-1}(f) - H^*(f)] \quad (15)$$

where:

$$O^*(f) = O(-f) \quad (16)$$

for real-valued object functions⁶. Using (12) we find:

$$\langle w(r_1, s_1, r_2, s_2) \rangle = (\text{const})_{s_{1z}, s_{2z}} \cdot \text{IFT}(\langle |N(f)_m|^2 \rangle) \cdot \langle O(f) \rangle \quad (17)$$

in which the average noise power spectrum is determined by (15) and IFT stands for the inverse Fourier transform. In the above, the average cross-spectral density of the outgoing field $\langle w(r_1, s_1, r_2, s_2, \omega) \rangle$ is taken in the entrance pupil of the imaging path. Since the interferometric path records the cross-spectral density in the exit pupil, one needs to link the cross-spectral densities in the entrance and the exit pupils. The relationship between cross-spectral densities in the two planes may be expressed in symbolic form by the convolution⁴:

$$w(1,2) = w^0(1,2) * [K(1,2) \cdot K^*(1,2)] \quad (18)$$

where $w^0(1,2)$ represents the cross-spectral density in the exit pupil for the pair of field points (1, 2) and $K(1,2)$ is the amplitude spread function of the imaging optics. Taking the average of (18) and substituting in (17) gives:

$$\langle w^0(1,2) * [K(1,2) \cdot K^*(1,2)] \rangle = (\text{const})_{s_{1z}, s_{2z}} \cdot \text{IFT}(\langle |N(f)_m|^2 \rangle) \cdot \langle O(f) \rangle \quad (19)$$

The above relationship forms the basis for object retrieval. The process may be completed via one of the following routes:

1) solving (19) for the average object spectrum and computing its inverse transform:

$$\langle o(x,y) \rangle = \text{IFT}(\langle O(f) \rangle) \quad (20)$$

2) using (19) as a closure relationship for matching the average object spectrum.

Either one of these approaches requires measurement of the average cross-spectral density in the exit pupil $\langle w^0(1,2) \rangle$. This is the topic of the next section.

3. THE PHASE CLOSURE METHOD

Pupil-plane interferometry is a high-resolution observation procedure for the restoration of images degraded by atmospheric turbulence^{8,9-11}. To overcome randomly induced errors on the fringe pattern, the interferometric baselines are arranged in

closed loops over the exit pupil plane. The closure phase is computed by adding the phases of detected fringes around these loops and it is shown to be free from additive distortions produced by turbulence and input optics^{9,10}. We consider below a three-aperture interferometer (fig. 3) with vector baselines \mathbf{b}_{12} , \mathbf{b}_{23} and \mathbf{b}_{31} . The fringe irradiance distribution near the optical axis of the interferometer has the following form¹⁰:

$$I(\mathbf{X}) = I_S(\mathbf{X}) \cdot [1 + |V_{ij}| \cdot \cos(2 \cdot \pi \cdot \mathbf{f}_{ij} \cdot \mathbf{X} + \phi_{ij})] \quad (21)$$

in which \mathbf{X} is a spatial coordinate having a parallel orientation to the baseline \mathbf{b}_{ij} , $I_S(\mathbf{X})$ is the sum of irradiances reaching the detector from the two apertures (i,j) (i,j = 1,2,3), V_{ij} represents the complex fringe visibility:

$$V_{ij} = |V_{ij}| \cdot \exp(j \cdot \phi_{ij}) \quad (22)$$

ϕ_{ij} is the fringe phase and \mathbf{f}_{ij} the vector spatial frequency defined by:

$$\mathbf{f}_{ij} = \mathbf{b}_{ij} / (\lambda \cdot R^0) \quad (23)$$

where R^0 represents the distance between the image of the object produced by the input/CCD optics and the exit pupil plane and λ is the average wavelength. For an isotropically emitting object it is reasonable to assume that each aperture is illuminated by equal irradiances. It can be shown that, in this case, the modulus of the complex fringe visibility is identical to the modulus of the normalized cross-spectral density⁴, i.e.:

$$|V_{ij}| = |w^0(i,j) / [w^0(i,i) \cdot w^0(j,j)]^{1/2}| \quad (24)$$

The phase-closure technique estimates the overall fringe visibility:

$$V_{123} = V_{12} \cdot V_{23} \cdot V_{31} = |V_{12}| \cdot |V_{23}| \cdot |V_{31}| \cdot \exp(j \cdot \phi_{123}) \quad (25)$$

in which the closure phase is given by:

$$\phi_{123} = \phi_{12} + \phi_{23} + \phi_{31} + \phi_{12}^n + \phi_{23}^n + \phi_{31}^n \quad (26)$$

In (26) ϕ_{ij}^n represents the detection noise contribution (assumed to be known) and:

$$V_{ij} = R_{ij} + j \cdot I_{ij} \quad (27)$$

$$\phi_{ij} = \text{atan}(I_{ij} / R_{ij})$$

where R_{ij} and I_{ij} are the real and imaginary components of the detected fringe signal¹⁰. Three points need to be made regarding the use of (26) and (27):

- phase unwrapping routines are required since ϕ_{ij} are known only modulo $2 \cdot \pi$.
- the quality of the fringe signal may be first increased by smoothing with low-pass Gaussian filters¹² or by applying suitable routines pertaining to fringe-pattern analysis¹³⁻¹⁹.
- the rotationally symmetric object irradiance implies that:

$$|V_{ij}| = |V_c| = \text{constant} \quad (28)$$

Up to a multiplicative constant, the average cross-spectral density in the exit pupil may be derived from (28), (27), (25) and (24) as:

$$\langle w^0(1,2) \rangle = \langle (\text{const}) \cdot (V_{123})^{1/3} \rangle = \langle (\text{const}) \cdot |V_c| \cdot \exp[j \cdot (\phi_{123} / 3)] \rangle \quad (29)$$

4. SUMMARY OF STEPS

In what follows we summarize the sequence of steps involved in the object retrieval strategy:

- 1) locate a potential target.
- 2) compute the CCD image spectrum with:

$$I(f) = FT[i(x,y)] \quad (30)$$

- 3) compute the modulus of the complex fringe visibility $|V_c|$ from (27):

$$|V_c| = (R_{ij}^2 + I_{ij}^2)^{1/2} \quad (31)$$

- 4) compute the closure phase ϕ_{123} from (26) and (27).
- 5) compute the average cross-spectral density in the exit pupil $\langle w^0(1,2) \rangle$ from (29).
- 6) compute the average cross-spectral density in the entrance pupil from (18).
- 7a) solve for the average object spectrum from (19) and (15).
- 7b) retrieve the average object irradiance with:

$$\langle o(x,y) \rangle = IFT(\langle O(f) \rangle) \quad (32)$$

or:

- 7c) match the average object spectrum based on (19) and (15).

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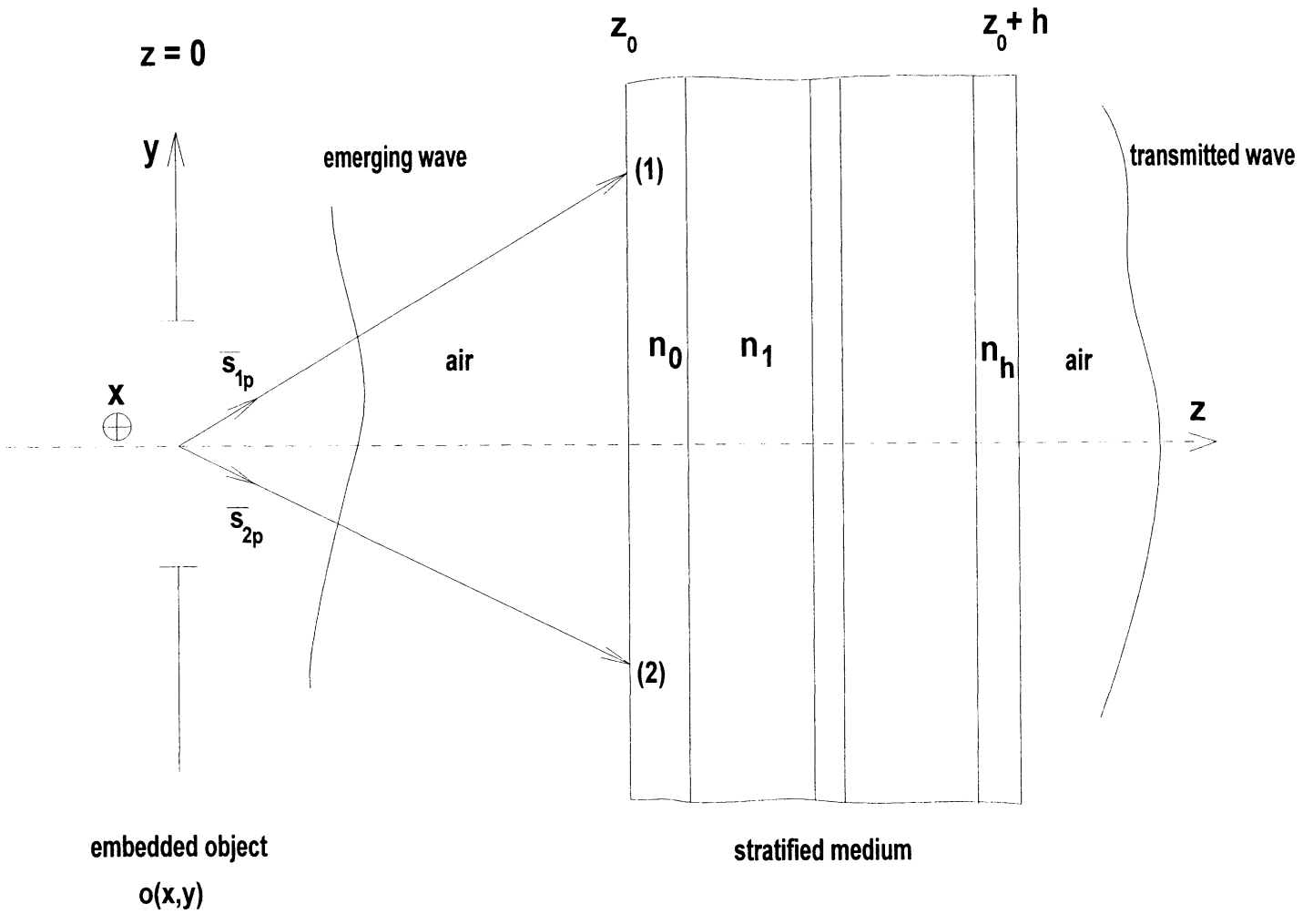


FIG. 1

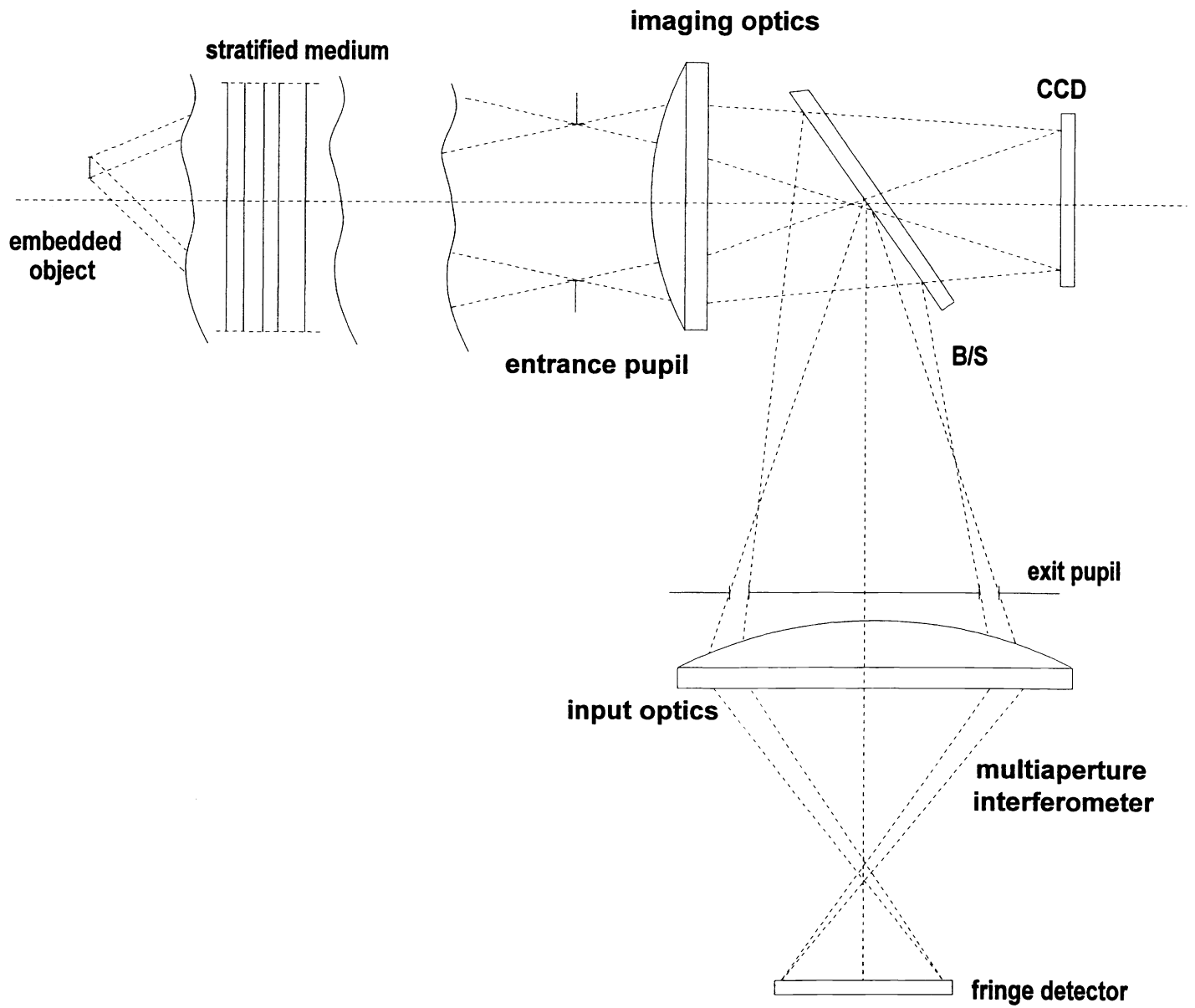


FIG. 2

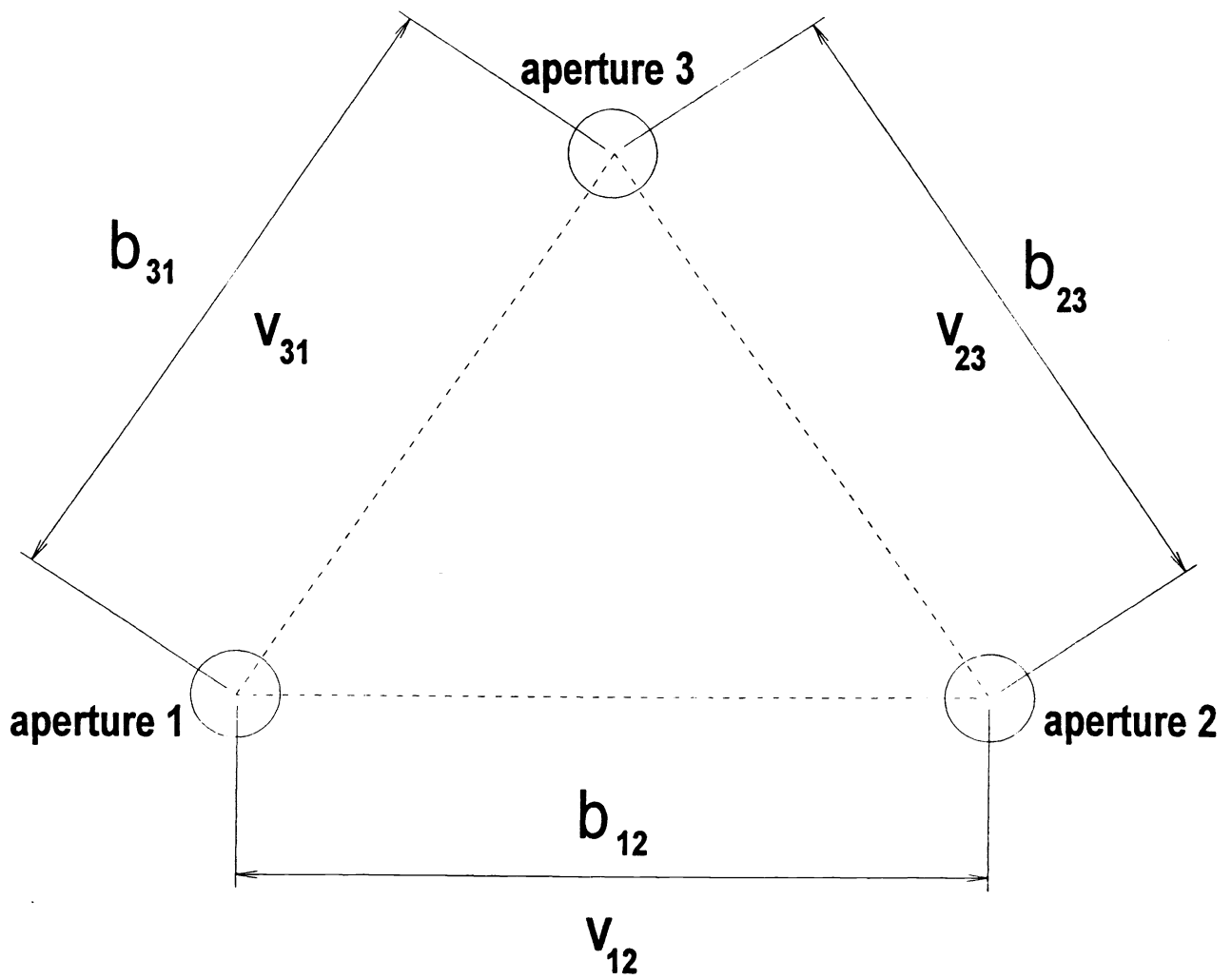


FIG. 3

