A Large Deviation Principle of Retarded Ornstein-Uhlenbeck Processes Driven by Lévy Noise

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Abstract: In this paper, we develop a large deviation principle (LDP) for a class of retarded Ornstein-Uhlenbeck processes driven by Lévy processes. We first present a LDP result for time delay systems driven by cylindrical Wiener processes based on the large deviations of Gaussian processes. By using a contraction technique and passing on a finite dimensional approximation, a large deviation principle is obtained for stochastic time delay evolution equations driven by additive Lévy noise, whose solutions are generally not Lévy processes any more.

Keywords: Retarded Ornstein-Uhlenbeck processes driven by Lévy noise; Large deviation principle; Lévy processes.

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1 Introduction

In practical applications, we remark that, in the finite dimensional case, time delay Ornstein-Uhlenbeck processes play an important role in various fields of research. For instance, this model has been applied to the study of a system consisting of a particle coupled to a delayed quartic potential in physiology [11], and a similar model to a stochastic system subjected to a time-delayed feedback loop that involves a sigmoidal conversion function in life sciences (see, [9], [17], [18], [24]). In these works, it may be that a proper infinite dimensional version as considered here, is more appropriate, as it can approximate in nature the real world due to the inclusion of spatial variables in the model.

On the other hand, there exists an extensive literature on large deviations of stochastic evolution equations, especially stochastic partial differential equations (SPDEs). For instance, we mention among others that Peszat [19] extended the large deviation principle of measures associated with finite-dimensional diffusions to measures given by a class of stochastic evolution equations with non-additive random perturbations, based on some exponential tail estimates for stochastic convolutions. In the case of parabolic SPDEs, Sower [21] proved a LDP in the set of α-Hölder continuous functions for $\alpha < 1/4$ when all coefficients of the equations are bounded and diffusion term is bounded away from zero. On the other hand, by focusing on specific SPDEs, large deviations for stochastic reaction-diffusion equations with non-Lipschitz reaction term are considered in [6]. The same problem is investigated in [5] for a class of Burgers' type SPDEs driven by the space-time white noise. Recently, the study of LDP for stochastic evolution equations driven by jump processes began to attract researchers' attention. For instance, Röcker and Zhang [20] developed a large deviation theory for infinite dimensional Ornstein-Uhlenbeck processes driven by Lévy processes.

The aim of this paper is to develop, in line with the spirit of [20], a LDP for the law of infinite dimensional time delay Ornstein-Uhlenbeck processes of retarded type. To this end, first let us state some notations and preliminary results.

Let H and K be two real separable Hilbert spaces with associated inner products $\langle\cdot,\cdot\rangle_H$, $\langle\cdot,\cdot\rangle_K$ and norms $\|\cdot\|_H$, $\|\cdot\|_K$, respectively. We denote by $\mathscr{L}(K,H)$ the set of all linear bounded operators from K into H, equipped with the usual operator norm $\|\cdot\|$. When $H = K$, we denote $\mathscr{L}(H,H)$ simply by $\mathscr{L}(H)$.

Throughout this work, we denote by $r > 0$ a fixed constant and define by $L_H^2 = L^2([-r,0];H)$ the space of all H-valued equivalent classes of measurable functions $\varphi(\theta)$, $\theta \in [-r,0]$, such that $\int_{-r}^0 \|\varphi(\theta)\|_H^2 d\theta < \infty$. Let $\mathcal{H}$ be the product space $H \times L_H^2$ with the norm defined by

$$\|\Phi\|_{\mathcal{H}} = (\|\phi_0\|_H^2 + \|\phi_1\|_{L_H^2}^2)^{1/2} \quad \text{for all} \quad \Phi = (\phi_0,\phi_1) \in \mathcal{H}.$$

Consider the following linear retarded differential equation on the Hilbert space H,

$$\begin{cases} dy(t) = Ay(t)dt + \Big(\int_{-r}^0 d\eta(\theta)y(t+\theta)\Big)dt \qquad \text{for any} \qquad t > 0, \\ y(0) = \phi_0,\ y(t) = \phi_1(t), \quad t \in [-r,0], \end{cases} \tag{1.1}$$

for arbitrarily given initial $\Phi = (\phi_0, \phi_1) \in \mathcal{H}$. Here A is the infinitesimal generator of some C_0-semigroup e^{tA}, $t \geq 0$, on H and η is given by the following Stieltjes measure

$$\eta(\tau) = -\mathbf{1}_{(-\infty,\,-r]}(\tau)A_1 - \int_\tau^0 A_0(\theta)d\theta, \qquad \tau \in [-r, 0], \tag{1.2}$$

where $\mathbf{1}_{(-\infty,-r]}(\tau)$ is the indicator function, $A_0(\cdot) \in L^2([-r, 0]; \mathscr{L}(H))$ and $A_1 \in \mathscr{L}(H)$. It is immediate to see that

$$\eta(\varphi) := \int_{-r}^0 d\eta(\theta)\varphi(\theta) = A_1\varphi(-r) + \int_{-r}^0 A_0(\theta)\varphi(\theta)d\theta, \quad \forall\, \varphi \in C([-r, 0]; H), \tag{1.3}$$

where $C([-r, 0]; H)$ is the space of all H-valued continuous functions on $[-r, 0]$. Moreover, we have the following result whose proof is referred to Lemma 5.1, [16] with a slight modification.

Lemma 1.1. *For arbitrary $T \geq 0$, the delay operator η defined in (1.3) permits a bounded linear extension, still denote it by η, from $L^2([-r, T]; H)$ into $L^2([0, T]; H)$. Moreover, there exists a real number $M > 0$ such that*

$$\int_0^T \Big\| \int_{-r}^0 d\eta(\theta)y(t + \theta)\Big\|_H^2 dt \leq M \int_{-r}^T \|y(t)\|_H^2 dt \quad \textit{for any} \quad y \in L^2([-r, T]; H) \tag{1.4}$$

where

$$M = \Big\{\|A_1\| + \|A_0(\cdot)\|_{L^2([-r,0];\mathscr{L}(H))} \cdot r^{1/2}\Big\}^2 > 0.$$

We define the so-called retarded Green operator $G(t)$, $t \in \mathbb{R}^1$, by the unique solution of the following operator integral equation

$$G(t) = \begin{cases} e^{tA} + \displaystyle\int_0^t e^{(t-s)A} \int_{-r}^0 d\eta(\theta)G(s + \theta)ds, & t \geq 0, \\ \mathrm{O}, & t < 0, \end{cases} \tag{1.5}$$

where O denotes the null operator on H. It may be shown (cf. [13]) that $G(\cdot)$ is a strongly continuous one-parameter family of bounded linear operators on H such that $\|G(t)\| \leq C \cdot e^{\gamma t}$, $t \geq 0$, for some constants $C > 0$ and $\gamma \in \mathbb{R}^1$.

For each function $\varphi : [-r, 0] \to H$, we define its right extension function $\vec{\varphi}$ through

$$\vec{\varphi} : [-r, \infty) \to H, \quad \vec{\varphi}(t) = \begin{cases} \varphi(t), & -r \leq t \leq 0, \\ 0, & 0 < t < \infty. \end{cases} \tag{1.6}$$

It is useful to introduce the following structure operator S on the space $C([-r, 0]; H)$ by

$$(S\varphi)(\theta) = \int_{-r}^0 d\eta(\tau)\vec{\varphi}(-\theta + \tau), \quad \theta \in [-r, 0], \quad \forall\, \varphi(\cdot) \in C([-r, 0]; H). \tag{1.7}$$

It may be shown (cf. [15]) that S is extendable to a linear and bounded operator, still denote it by S, from $L^2([-r, 0]; H)$ or $L^2([-r, 0]; \mathscr{L}(H))$, into itself, respectively. In general, the family $G(t)$, $t \in \mathbb{R}^1$, would no longer be a semigroup on H. However, it may be shown that it is a "quasi-semigroup" in the sense that

$$G(t+s)x = G(t)G(s)x + \int_{-r}^{0} G(t+\theta)[SG(s+\cdot)](\theta)x d\theta \qquad \text{for all} \quad s,\ t \geq 0, \quad x \in H. \tag{1.8}$$

Let $\{\Omega, \mathscr{F}, \mathbb{P}\}$ be a complete probability space equipped with some filtration $\{\mathscr{F}_t\}_{t\geq 0}$. Let $\{W(t),\ t \geq 0\}$ denote a K-valued $\{\mathscr{F}_t\}_{t\geq 0}$-Wiener process defined on $\{\Omega, \mathscr{F}, \mathbb{P}\}$ with covariance operator Q, i.e.,

$$\mathbb{E}\langle W(t), x\rangle_K \langle W(s), y\rangle_K = (t \wedge s)\langle Qx, y\rangle_K \quad \text{for all} \quad x,\ y \in K, \quad s,\ t \in [0, \infty),$$

where Q is a linear, symmetric and nonnegative bounded operator on K. In particular, we shall call $W(t)$, $t \geq 0$, a K-valued Q-Wiener process with respect to $\{\mathscr{F}_t\}_{t\geq 0}$. If the trace $\operatorname{Tr} Q < \infty$, then W is a genuine Wiener process. If the trace $\operatorname{Tr} Q = \infty$, then W is called a cylindrical Wiener process. On the other hand, let $(X, \sigma(X), \nu)$ be a σ-finite measurable space. Let $p = (p(t))$, $t \geq 0$, be some stationary $\{\mathscr{F}_t\}$-Poisson point process on X with characteristic measure ν (cf. [12]). Let $N(dt, dx)$ be the Poisson counting measure associated with the process $p = (p(t))$ and we define the compensated Poisson measure

$$\tilde{N}(dt, dx) := N(dt, dx) - dt\nu(dx) \tag{1.9}$$

so that $\tilde{N}([0, t], E)$, $t \geq 0$, $E \in \sigma(X)$, turns out to be a martingale measure.

In this work we shall consider the following retarded Ornstein-Uhlenbeck type stochastic evolution equation driven by Lévy processes on the Hilbert space H,

$$\begin{cases} dy(t) = Ay(t)dt + \displaystyle\int_{-r}^{0} d\eta(\theta)y(t+\theta)dt + dL(t), \\ y(0) = \phi_0,\ \ y(t) = \phi_1(t), \ \ t \in [-r, 0], \end{cases} \tag{1.10}$$

for some proper initial data $\Phi = (\phi_0, \phi_1)$, where $L(t)$, $t \geq 0$, is a Lévy process given by

$$L(t) = bt + W(t) + \int_0^t \int_X J(x)\tilde{N}(ds, dx), \qquad t \geq 0. \tag{1.11}$$

Here b is a constant vector in H, J is a measurable mapping from X into H and W is an H-valued Q-Wiener process. We are interested in developing a large deviation principle for the equation (1.10). More precisely, consider the stochastic retarded evolution equation

$$\begin{cases} y^n(t) = \phi_0 + \displaystyle\int_0^t Ay^n(s)ds + \int_0^t \int_{-r}^{0} d\eta(\theta)y^n(s+\theta)ds + bt + \frac{W(t)}{\sqrt{n}} \\ \qquad\qquad + \displaystyle\frac{1}{n}\int_0^t \int_X J(x)\tilde{N}_n(ds, dx), \quad t \geq 0, \\ y^n(0) = \phi_0,\ \ y^n(t) = \phi_1(t),\ \ t \in [-r, 0],\ \ \Phi = (\phi_0, \phi_1),\ \ n \in \mathbb{N}, \end{cases} \tag{1.12}$$

where $\tilde{N}_n(ds, dx)$ denotes the compensated Poisson measure with intensity measure $n\nu$. The main purpose is to establish a large deviation principle for the law $\mu_n(\cdot)$ of the solutions

$y^n(t)$, $t \geq 0$, of (1.12) on $D([0,T];H)$, the space of all cadlag paths from $[0,T]$ into H, for any fixed $T \geq 0$.

The organization of this work is as follows. In Section 2, we focus on a class of time delay systems driven by an additive white noise in which the associated Wiener process could be a cylindrical Wiener process. We present a LDP of this system based on the existing large deviation results for infinite dimensional Gaussian processes. Section 3 is devoted to the establishment of some useful results for deterministic time delay systems, which will play a key role in the subsequent large deviation analysis. In Section 4, by using a contraction technique and passing on a finite dimensional approximation, we shall establish a large deviation principle for a class of retarded Ornstein-Uhlenbeck processes driven by additive Lévy noise.

2 LDP of Stochastic Systems Driven by White Noise

Let $n \in \mathbb{N}$ and $T \geq 0$. We shall consider mild solutions of the following stochastic retarded differential equations on the Hilbert space H,

$$\begin{cases} y^n(t) = \phi_0 + \int_0^t Ay^n(s)ds + \int_0^t \int_{-r}^0 d\eta(\theta)y^n(s+\theta)ds + \frac{1}{\sqrt{n}}BW(t) \quad \text{for any} \quad t \in [0,T], \\ y^n(0) = \phi_0, \;\; y^n(t) = \phi_1(t), \quad t \in [-r,0], \;\; \Phi = (\phi_0, \phi_1) \in \mathcal{H}, \end{cases} \tag{2.1}$$

where A generates a C_0-semigroup e^{tA}, $t \geq 0$, $B \in \mathscr{L}(K,H)$ and $W(t)$, $t \geq 0$, is a K-valued Q-Wiener process. For arbitrary $t \geq 0$, define $Q_t = \int_0^t G(s)BQB^*G^*(s)dt$. It may be shown (cf. [13]) that if $\operatorname{Tr} Q_t < \infty$ for each $t \in [0,T]$, then the equation (2.1), for each $n \in \mathbb{N}$, has a unique mild solution which is represented by

$$y^n(t) = G(t)\phi_0 + \int_{-r}^0 \int_{-r}^\theta G(t-\theta+\tau)d\eta(\tau)\phi_1(\theta)d\theta + \frac{1}{\sqrt{n}} \int_0^t G(t-s)BdW(s), \;\; t \in [0,T].$$

To proceed further, let us first consider the stochastic convolution process

$$W_G(t) = \int_0^t G(t-s)BdW(s), \qquad t \in [0,T].$$

Lemma 2.1. *For arbitrary $T \geq 0$, the law $\mu(W_G(\cdot))$ is a symmetric Gaussian measure on $L^2([0,T];H)$ with the covariance operator R given by*

$$R\xi(t) = \int_0^T r(t,s)\xi(s)ds, \qquad \forall\, \xi \in L^2([0,T];H) \tag{2.2}$$

where

$$r(t,s) = \int_0^{t\wedge s} G(t-v)BQB^*G^*(s-v)dv, \tag{2.3}$$

and $t \wedge s = \min\{t,s\}$.

Proof. It is evident to see that $W_G(\cdot)$ could be regarded as an $L^2([0,T];H)$-valued random variable. It is also immediate that the law $\mu(W_G(\cdot))$ is symmetric and Gaussian on $L^2([0,T];H)$ by using Proposition 2.9, p. 42 and Lemma 5.2, p. 121 in [2].

To show (2.2) and (2.3), we notice that for any $\xi,\ \zeta \in L^2([0,T];H)$, by definition,

$$\begin{aligned}\langle R\xi,\zeta\rangle_{L^2([0,T];H)} &= \mathbb{E}\Big(\int_0^T \langle \xi(t), W_G(t)\rangle_H dt \int_0^T \langle \zeta(s), W_G(s)\rangle_H ds\Big)\\ &= \int_0^T\int_0^T \mathbb{E}\langle \xi(t), W_G(t)\rangle_H \langle \zeta(s), W_G(s)\rangle_H dtds.\end{aligned} \tag{2.4}$$

On the other hand, for any $t > s \geq 0$, we have by virtue of (1.8) and $G(t) = \mathrm{O}$ for $t < 0$ that

$$\begin{aligned}
&\mathbb{E}(\langle \xi(t), W_G(t)\rangle_H \langle \zeta(s), W_G(s)\rangle_H)\\
&= \mathbb{E}\Big(\Big\langle \xi(t), \int_0^t G(t-v)BdW(v)\Big\rangle_H \langle \zeta(s), W_G(s)\rangle_H\Big)\\
&= \mathbb{E}\Big[\Big\langle \xi(t), \int_0^t \Big[G(t-s)G(s-v) + \int_{-r}^0 G(t-s+\theta)[SG(s-v+\cdot)](\theta)d\theta\Big]BdW(v)\Big\rangle_H\\
&\qquad\qquad \cdot \langle \zeta(s), W_G(s)\rangle_H\Big]\\
&= \mathbb{E}\Big[\Big\langle \xi(t), G(t-s)\int_0^s G(s-v)BdW(v)\\
&\quad + \int_0^t\int_{-r}^0 G(t-s+\theta)[SG(s-v+\cdot)](\theta)Bd\theta dW(v)\Big\rangle_H \Big\langle \zeta(s), \int_0^s G(s-v)BdW(v)\Big\rangle_H\Big].
\end{aligned} \tag{2.5}$$

Note that the equality (1.8) yields the following dual relation

$$G^*(t+s) = G^*(s)G^*(t) + \int_{-r}^0 G^*(s+\theta)[S^*G^*(t+\cdot)](\theta)d\theta \qquad \text{for all} \quad s,\ t \geq 0. \tag{2.6}$$

This further implies, in addition to (2.5), that

$$\begin{aligned}
&\mathbb{E}(\langle \xi(t), W_G(t)\rangle_H \langle \zeta(s), W_G(s)\rangle_H)\\
&= \mathbb{E}\Big[\Big(\Big\langle G^*(t-s)\xi(t), \int_0^s G(s-v)BdW(v)\Big\rangle_H\\
&\quad + \int_{-r}^0 \Big\langle [S^*G^*(t-s+\cdot)](\theta)\xi(t), \int_0^s G(s-v+\theta)BdW(v)\Big\rangle_H d\theta\Big)\\
&\qquad\qquad \cdot \Big\langle \zeta(s), \int_0^s G(s-v)BdW(v)\Big\rangle_H\Big]\\
&= \Big\langle \int_0^s G(s-v)BQB^*G^*(s-v)G^*(t-s)dv\xi(t)\\
&\quad + \int_0^s G(s-v)BQB^*\int_{-r}^0 G^*(s-v+\theta)[S^*G^*(t-s+\cdot)](\theta)d\theta dv\xi(t), \zeta(s)\Big\rangle_H\\
&= \Big\langle \int_0^s G(s-v)BQB^*G^*(t-v)dv\xi(t), \zeta(s)\Big\rangle_H.
\end{aligned}$$

Hence, the desired results follows and the proof is complete. □

For any probability measure μ on $L^2([0,T];H)$, we define a family of measures $\{\mu_n\}_{n\in\mathbb{N}}$ by

$$\mu_n(\Gamma) := \mu\Big(\frac{1}{\sqrt{n}}\Gamma\Big), \qquad \Gamma \in \mathscr{B}(L^2([0,T];H)), \qquad n \in \mathbb{N}, \tag{2.7}$$

where $\mathscr{B}(L^2([0,T];H))$ is the Borel σ-field on $L^2([0,T];H)$. We first recall the following LDP results of Gaussian measures (cf. [2]).

Proposition 2.1. *For any $T \geq 0$, assume that μ is the Gaussian measure $N(0,R)$ on the Hilbert space $L^2([0,T];H)$. Then the family $\{\mu_n\}_{n\geq 1}$ given by (2.7) satisfies a LDP with the rate function*

$$I(z) = \begin{cases} \frac{1}{2}\|R^{-1/2}z\|^2_{L^2([0,T];H)} & \text{if} \quad z \in Ran\, R^{1/2}, \\ \infty & \text{otherwise}, \end{cases}$$

where $Ran\, R^{1/2}$ is the range of operator $R^{1/2}$.

To proceed further, let us consider the following deterministic control system on the Hilbert space H,

$$\begin{cases} dy(t) = Ay(t)dt + \int_{-r}^0 d\eta(\theta)y(t+\theta)dt + BQ^{1/2}u(t)dt, \quad t \in [0,T], \\ y(0) = \phi_0, \ y(t) = \phi_1(t), \ t \in [-r,0], \ \Phi = (\phi_0,\phi_1) \in \mathcal{H}, \end{cases} \tag{2.8}$$

where $T \geq 0$ and $u \in L^2([0,T];K)$. It is easy to see that the explicit solution $y^{\Phi,u}$ of (2.8) is given by

$$y^{\Phi,u}(t) = G(t)\phi_0 + \int_{-r}^0\int_{-r}^\theta G(t-\theta+\tau)d\eta(\tau)\phi_1(\theta)d\theta + \int_0^t G(t-s)BQ^{\frac{1}{2}}u(s)ds, \ t \in [0,T].$$

For any $T \geq 0$, define a mapping $\mathcal{L} : L^2([0,T];K) \to L^2([0,T];H)$ by

$$\mathcal{L}u(t) = \int_0^t G(t-s)BQ^{1/2}u(s)ds, \qquad u \in L^2([0,T];K),$$

and it is easy to see that

$$(\mathcal{L}^*y)(t) = \int_t^T Q^{1/2}B^*G^*(s-t)y(s)ds, \qquad y \in L^2([0,T];H).$$

Let $\mathcal{R} = \mathcal{L}\mathcal{L}^* : \ L^2([0,T];H) \to L^2([0,T];H)$, then it is immediate to have that

$$(\mathcal{R}y)(t) = \int_0^T r(t,s)y(s)ds, \qquad t \in [0,T], \tag{2.9}$$

where

$$r(t,s) = \int_0^{t\wedge s} G(t-v)BQB^*G^*(s-v)dv, \qquad 0 \leq s, \ t \leq T. \tag{2.10}$$

The following proposition whose proofs are referred to p. 411, [2] is useful in establishing our LDP results.

Proposition 2.2. *For arbitrary $T \geq 0$, it holds true that*

$$Ran\,\mathcal{L} = Ran\,\mathcal{R}^{1/2}$$

and for any $z \in Ran\,\mathcal{R}^{1/2} \subset L^2([0,T];H)$, there is

$$\|\mathcal{R}^{-\frac{1}{2}}z\|^2_{L^2([0,T];H)} = \inf\Big\{ \int_0^T \|Q^{-\frac{1}{2}}u(t)\|^2_K dt : \ u(t) \in Ran\,Q^{\frac{1}{2}},\ Q^{-\frac{1}{2}}u(t) \in L^2([0,T];K) \\ \text{such that } \int_0^t G(t-s)Bu(s)ds = z(t),\ t \in [0,T]\Big\}.$$

Now we are in a position to state the main result in this section.

Theorem 2.1. *For any $T \geq 0$, the laws $\{\mu_n\}_{n\geq 1}$ of the solution $\{y^n(\cdot)\}_{n\geq 1}$ of (2.1), defined in (2.7), satisfy a LDP on $L^2([0,T];H)$ with the associated rate functional*

$$I(z) = \begin{cases} \inf\Big\{\frac{1}{2}\int_0^T \|Q^{-\frac{1}{2}}u(t)\|^2_K dt : u(t) \in Ran\,Q^{\frac{1}{2}},\ Q^{-\frac{1}{2}}u(t) \in L^2([0,T];K) \\ \qquad \text{such that } \int_{-r}^0\int_{-r}^\theta G(t-\theta+\tau)d\eta(\tau)\phi_1(\theta)d\theta \\ \qquad + G(t)\phi_0 + \int_0^t G(t-s)Bu(s)ds = z(t),\ t \in [0,T]\Big\}, \\ \infty \qquad \text{otherwise,} \end{cases} \tag{2.11}$$

for any $z \in L^2([0,T];H)$.

Proof. It suffices to prove this result for $\Phi = (0,0)$. In this case, note that by Lemma 2.1, the law $\mu_n(\cdot)$ is a symmetric Gaussian measure on $L^2([0,T];H)$ for each $n \in \mathbb{N}$ with covariance operator $R = \mathcal{R}$ given by (2.9) and (2.10). Hence, the conclusion follows from Propositions 2.1 and 2.2. □

3 Some Useful Results

In the remainder of this work, we shall establish a LDP for the system (1.12). Because we confine ourselves, in this case, to additive Lévy noise. The method employed for Gaussian case in the last section does not work. The reason is that the solution of (1.12) is no longer a Lévy process on this occasion. The additive noise case is already quite involved.

Let V be a Hilbert space with norm $\|\cdot\|_V$ which is embedded in H. We identity H with its dual space H^* and denote the dual of V by V^*. Then we have the relation

$$V \hookrightarrow H \cong H^* \hookrightarrow V^*$$

where the inclusions $\hookrightarrow$ are assumed to be dense and continuous so that for some constant $\beta > 0$,

$$\|v\|_H^2 \le \beta \|v\|_V^2 \quad \text{for all} \quad v \in V. \tag{3.1}$$

If we denote the dual pair between V and V^* by $\langle \cdot, \cdot \rangle_{V,V^*}$, it is clear that

$$\langle u, v \rangle_{V,V^*} = \langle u, v \rangle_H \ \text{ for all } \ u \in V, \ v \in H.$$

Let $a(u, v)$ be a bounded sesquilinear form defined in $V \times V$ satisfying Gårding's inequality

$$2a(u, u) \ge \alpha \|u\|_V^2 - \lambda \|u\|_H^2, \qquad u \in V, \tag{3.2}$$

where $\alpha > 0$ and $\lambda \ge 0$ are constants. Let A be the operator associated with this sesquilinear form

$$\langle v, Au \rangle_{V,V^*} = -a(u, v), \quad u, \ v \in V. \tag{3.3}$$

The operator A is bounded and linear from V into V^*. The realization of A in H, which is the restriction of A to the domain $\mathscr{D}(A) = \{v \in V : Av \in H\}$ is also denoted by A. It is well known (cf. [22]) that A generates an analytic semigroup e^{tA}, $t \ge 0$, in H.

Let $L_V^2 = L^2([-r, 0]; V)$ be the space of all V-valued equivalent classes of measurable functions $\varphi(\theta)$, $\theta \in [-r, 0]$, such that $\int_{-r}^0 \|\varphi(\theta)\|_V^2 d\theta < \infty$. Let $\mathcal{V}$ be the product space $H \times L_V^2$ with the norm defined by

$$\|\Phi\|_{\mathcal{V}} = (\|\phi_0\|_H^2 + \|\phi_1\|_{L_V^2}^2)^{1/2} \ \text{ for all } \ \Phi = (\phi_0, \phi_1) \in \mathcal{V}.$$

Let $\Phi = (\phi_0, \phi_1) \in \mathcal{V}$ and $T \ge 0$. For arbitrarily given $f \in L^2([0, T]; V)$, we define $y(f) = y(\cdot, f) \in L^2([-r, T]; V)$ as the unique solution to the following equation:

$$\begin{cases} y(t, f) = \phi_0 + \displaystyle\int_0^t Ay(s, f)ds + \int_0^t \int_{-r}^0 d\eta(\theta) y(s + \theta, f)ds + f(t), \qquad t \in [0, T], \\ y(0, f) = \phi_0 \in H, \ \ y(t, f) = \phi_1(t) \in L^2([-r, 0]; V), \qquad t \in [-r, 0]. \end{cases} \tag{3.4}$$

For any $T \ge 0$, let $D([0, T]; V)$ be the space of all cadlag mappings from $[0, T]$ into V.

Lemma 3.1. *Let $T \ge 0$. Then the mapping $y(\cdot)$ defined by the equation (3.4) is continuous from $D([0, T]; V)$ into $D([-r, T]; H) \cap L^2([-r, T]; V)$ in the topology of uniform convergence.*

Proof. For any $f \in L^2([0, T]; V)$, we may define an extension $\bar{f}(t) = f(t)$ for $t \in [0, T]$ and $\bar{f}(t) = 0$ for $t \in [-r, 0]$. Let us put

$$x(f)(t) = x(t, f) := y(t, f) - \bar{f}(t) \ \text{ for all } \ t \in [-r, T]$$

and substitute this into (3.4). Then it is easy to see that $x(\cdot, f)$ satisfies the following integral equation:

$$\begin{cases} x(t, f) = \phi_0 + \displaystyle\int_0^t Ax(s, f)ds + \int_0^t Af(s)ds + \int_0^t \int_{-r}^0 d\eta(\theta) x(s + \theta, f)ds \\ \qquad\qquad\qquad\qquad\qquad\qquad + \displaystyle\int_0^t \int_{-r}^0 d\eta(\theta) \bar{f}(s + \theta)ds, \qquad t \in [0, T], \\ x(0, f) = \phi_0 \in H, \quad x(t, f) = \phi_1(t) \in L^2([-r, 0]; V), \quad t \in [-r, 0]. \end{cases} \tag{3.5}$$

Hence, to establish the desired result, it suffices to show that the mapping

$$x(\cdot)(t):\ D([0,T];V)\to D([0,T];H)\cap L^2([0,T];V),\qquad t\in[0,T],$$

is continuous.

Let $f_n,\ f\in D([0,T];V)$ with respective extensions $\bar{f}_n,\ \bar{f}\in D([-r,T];V)$ as above such that $f_n\to f$ in $D([0,T];V)$ as $n\to\infty$. Then we have by the chain rule that for any $t\in[0,T]$,

$$\begin{aligned}
&\|x(t,f_n)-x(t,f)\|_H^2\\
&=2\int_0^t\langle x(s,f_n)-x(s,f),A(x(s,f_n)-x(s,f))\rangle_{V,V^*}ds\\
&\quad+2\int_0^t\langle x(s,f_n)-x(s,f),A(f_n-f)(s)\rangle_{V,V^*}ds\\
&\quad+2\int_0^t\Big\langle\int_{-r}^0 d\eta(\theta)\Big(x(s+\theta,f_n)-x(s+\theta,f)\Big),x(s,f_n)-x(s,f)\Big\rangle_H ds\\
&\quad+2\int_0^t\Big\langle\int_{-r}^0 d\eta(\theta)\Big(\bar{f}_n(s+\theta)-\bar{f}(s+\theta)\Big),x(s,f_n)-x(s,f)\Big\rangle_H ds
\end{aligned}\tag{3.6}$$

which, together with (3.2), (3.3) and (1.4), implies immediately that for $t\in[0,T]$,

$$\begin{aligned}
&\|x(t,f_n)-x(t,f)\|_H^2\\
&\le-\alpha\int_0^t\|x(s,f_n)-x(s,f)\|_V^2ds+\lambda\int_0^t\|x(s,f_n)-x(s,f)\|_H^2ds\\
&\quad+2\int_0^t\sqrt{\alpha/2}\|x(s,f_n)-x(s,f)\|_V\cdot\sqrt{2/\alpha}\|A(f_n-f)(s)\|_{V^*}ds\\
&\quad+2\int_0^t\|x(s,f_n)-x(s,f)\|_H\Big\|\int_{-r}^0 d\eta(\theta)\Big(x(s+\theta,f_n)-x(s+\theta,f)\Big)\Big\|_H ds\\
&\quad+2\int_0^t\|x(s,f_n)-x(s,f)\|_H\cdot\Big\|\int_{-r}^0 d\eta(\theta)\Big(\bar{f}_n(s+\theta)-\bar{f}(s+\theta)\Big)\Big\|_H ds\\
&\le-\frac{\alpha}{2}\int_0^t\|x(s,f_n)-x(s,f)\|_V^2ds+(\lambda+M+2)\int_0^t\|x(s,f_n)-x(s,f)\|_H^2ds\\
&\quad+\frac{2\|A\|_{V,V^*}^2}{\alpha}\int_0^t\|f_n(s)-f(s)\|_V^2ds+M\int_0^t\|f_n(s)-f(s)\|_H^2ds.
\end{aligned}\tag{3.7}$$

In view of (3.1), this implies further that for any $t\in[0,T]$,

$$\begin{aligned}
&\|x(t,f_n)-x(t,f)\|_H^2+\frac{\alpha}{2}\int_0^t\|x(s,f_n)-x(s,f)\|_V^2ds\\
&\le(\lambda+M+2)\int_0^t\|x(s,f_n)-x(s,f)\|_H^2ds+\Big(\frac{2\|A\|_{V,V^*}^2}{\alpha}+M\beta\Big)\int_0^t\|f_n(s)-f(s)\|_V^2ds.
\end{aligned}\tag{3.8}$$

By virtue of Gronwall's inequality, it is easy to derive the desired continuity of the mapping $x(\cdot)$ (and thus $y(\cdot)$), so that the proof is complete. □

To proceed further, we first establish a lemma which is of its own importance.

Lemma 3.2. *Suppose $K(\cdot) \in L^\infty([0,T];\mathscr{L}(H))$ for each $T \geq 0$. Suppose that $S(t)$, $t \geq 0$, is a C_0-semigroup which is compact for all $t > 0$ on H, then*

(i) the bounded operator $F(t) = \int_0^t S(t-s)K(s)ds$ is compact for all $t > 0$. In particular, the retarded Green operator $G(t)$ is compact for all $t > 0$;

(ii) the bounded operator $H(t) = \int_0^t G(t-s)K(s)ds$ is compact for all $t > 0$.

Proof. (i) Note that we can write $F(t)$ as

$$F(t) = S(\varepsilon)F(t-\varepsilon) + \int_{t-\varepsilon}^t S(t-s)K(s)ds \quad \text{for any} \;\; \varepsilon \in (0,t].$$

By the compactness of $S(\varepsilon)$, $\varepsilon > 0$, and the boundedness of $F(t-\varepsilon)$, $S(\varepsilon)F(t-\varepsilon)$ is compact. Moreover, it is easy to see that

$$\Big\| \int_{t-\varepsilon}^t S(t-s)K(s)ds \Big\| \leq \sup_{s\in[0,t]} \|K(s)\| \int_0^\varepsilon \|S(s)\|ds \to 0 \;\; \text{as} \;\; \varepsilon \to 0.$$

Hence, $F(t)$ is also compact as a uniform limit of compact operators.

For the compactness of $G(t)$, let $S(t)$, $t \geq 0$, be the C_0-semigroup with its infinitesimal generator A. In this case, note that $G(t)$ satisfies

$$G(t) = \begin{cases} S(t) + \displaystyle\int_0^t \int_{-r}^0 S(t-s)d\eta(\theta)G(s+\theta)ds, & t \geq 0, \\ \mathrm{O}, & t < 0, \end{cases}$$

and $\int_{-r}^0 d\eta(\theta)G(\cdot+\theta) \in L^\infty([0,t];\mathscr{L}(H))$ for each $t > 0$. Thus the compactness of $S(t)$ implies the compactness of $G(t)$ for all $t > 0$.

(ii) On this occasion, we can write $H(t)$ for any $\varepsilon \in (0,t]$ as

$$\begin{aligned} H(t) = G(\varepsilon)F(t-\varepsilon) &+ \int_{t-\varepsilon}^t G(t-s)K(s)ds \\ &+ \int_0^{t-\varepsilon} \int_{-r}^0 G(t-s-\varepsilon+\theta)[SG_\varepsilon](\theta)K(s)d\theta ds, \end{aligned}$$

where S is the structure operator introduced in (1.7). By the compactness of $G(\varepsilon)$, $\varepsilon > 0$, and the boundedness of $F(t-\varepsilon)$, $G(\varepsilon)F(t-\varepsilon)$ is compactness. Moreover, it is easy to see that

$$\Big\| \int_{t-\varepsilon}^t G(t-s)K(s)ds \Big\| \leq \sup_{s\in[0,t]} \|K(s)\| \int_0^\varepsilon \|G(s)\|ds \to 0 \;\; \text{as} \;\; \varepsilon \to 0,$$

and, by the boundedness of the structure operator S on $L^2([-r,0];\mathscr{L}(H))$ and Hölder's inequality, we have

$$\begin{aligned}\Big\|\int_0^{t-\varepsilon}\int_{-r}^0 G(t-s-\varepsilon+\theta)[SG_\varepsilon](\theta)K(s)d\theta ds\Big\|\\ &\le t\sup_{s\in[0,t]}\|K(s)\|\|G(s)\|\int_{-r}^0\|SG_\varepsilon(\theta)\|d\theta\\ &\le r^{1/2}t\sup_{s\in[0,t]}\|K(s)\|\|G(s)\|\Big(\int_{-r}^0\|[SG_\varepsilon](\theta)\|^2d\theta\Big)^{1/2}\\ &\le r^{1/2}t\sup_{s\in[0,t]}\|K(s)\|\|G(s)\|\|S\|^{1/2}\Big(\int_0^\varepsilon\|G(\tau)\|^2d\tau\Big)^{1/2}\to 0\quad\text{as}\quad\varepsilon\to 0.\end{aligned}\tag{3.9}$$

Hence, $H(t)$ is also compact as a uniform limit of compact operators. The proof of the lemma is complete. □

Let $T\ge 0$. For any fixed $f\in L^1([0,T];H)$, we define an operator $\mathcal{K}$ by

$$\mathcal{K}f(t)=\int_0^t G(t-s)f(s)ds,\quad t\in[0,T],\tag{3.10}$$

which is the mild solution of the deterministic evolution equation:

$$\begin{cases}y(t)=\displaystyle\int_0^t Ay(s)ds+\int_0^t\int_{-r}^0 d\eta(\theta)y(s+\theta)ds+\int_0^t f(s)ds, & t\in[0,T],\\ y(0)=0,\ y(t)=0,\ t\in[-r,0].\end{cases}\tag{3.11}$$

Proposition 3.1. *Suppose that the C_0-semigroup e^{tA} generated by A is compact for each $t>0$. Let $T\ge 0$ and assume that $\mathcal{G}\subset L^1([0,T];H)$ is uniformly integrable, then the set $\mathcal{K}(\mathcal{G})$ is relatively compact in $C([0,T];H)$.*

Proof. To establish this proposition, we only need to show, according to Ascoli-Arzelà theorem, that:

(i) for each $t\in[0,T]$, the set $\{\mathcal{K}f(t);\ f\in\mathcal{G}\}$ is relatively compact in H;

(ii) for each $\varepsilon>0$, there exists a $\delta>0$ such that if $0\le s\le t\le T$, $t-s\le\delta$, then

$$\|\mathcal{K}f(t)-\mathcal{K}f(s)\|_H\le\varepsilon\quad\text{for all}\quad f\in\mathcal{G}.$$

We first prove the claim (i). For any $f\in\mathcal{G}\subset L^2([0,T];H)$ and fixed $t\in(0,T]$, the quasi-semigroup relation (1.8) implies that for any $\varepsilon\in(0,t]$,

$$\begin{aligned}\int_0^t G(t-s)f(s)ds&=G(\varepsilon)\int_0^{t-\varepsilon}G(t-\varepsilon-s)f(s)ds+\int_{t-\varepsilon}^t G(t-s)f(s)ds\\ &\quad+\int_0^{t-\varepsilon}\int_{-r}^0 G(t-s-\varepsilon+\theta)[SG_\varepsilon](\theta)f(s)d\theta ds\\ &=:I_1(\varepsilon,t,f)+I_2(\varepsilon,t,f)+I_3(\varepsilon,t,f).\end{aligned}\tag{3.12}$$

Since $G(\varepsilon)$, $\varepsilon > 0$, is compact in accordance with Lemma 3.2, $\{I_1(\varepsilon, t, f),\ f \in \mathcal{G}\}$ is relatively compact in H for $\varepsilon > 0$. On the other hand, since $\mathcal{G}$ is uniformly integrable, it is easy to see that

$$\begin{aligned}\sup_{f\in\mathcal{G}} \|I_2(\varepsilon, t, f)\|_H &\le \sup_{t\in[0,T]} \|G(t)\| \sup_{f\in\mathcal{G}} \int_{t-\varepsilon}^{t} \|f(s)\|_H ds\\ &\to 0 \quad \text{as} \quad \varepsilon \to 0.\end{aligned}$$

Lastly, for the term $I_3(\varepsilon, t, f)$, $f \in \mathcal{G}$, and $t \in (0, T]$, we have

$$\begin{aligned}\sup_{f\in\mathcal{G}} \|I_3(\varepsilon, t, f)\|_H &\le \sup_{f\in\mathcal{G}} \int_0^{t-\varepsilon} \|f(s)\|_H ds \int_{-r}^{0} \|G(t-s-\varepsilon+\theta)\| \cdot \|SG_\varepsilon(\theta)\| d\theta\\ &\le \sup_{t\in[0,T]} \|G(t)\| \sup_{f\in\mathcal{G}} \int_0^{t} \|f(s)\|_H ds \int_{-r}^{0} \|SG_\varepsilon(\theta)\| d\theta\\ &\le \|S\|^2 r \sup_{t\in[0,T]} \|G(t)\| \sup_{f\in\mathcal{G}} \int_0^{T} \|f(s)\|_H ds \int_0^{\varepsilon} \|G(s)\|^2 ds\\ &\to 0 \quad \text{as} \quad \varepsilon \to 0.\end{aligned} \tag{3.13}$$

Next, we show the second claim (ii). For any $t \in [0, T)$ and $\delta > 0$ with $t + \delta < T$, we have

$$\begin{aligned}\|\mathcal{K}f(t+\delta) - \mathcal{K}f(t)\|_H &\le \int_0^t \|G(t+\delta-s) - G(t-s)\| \|f(s)\|_H ds\\ &\quad + \int_t^{t+\delta} \|G(t+\delta-s)\| \|f(s)\|_H ds\\ &=: J_1(\delta, t, f) + J_2(\delta, t, f).\end{aligned} \tag{3.14}$$

Let $\kappa = \sup_{t\in[0,T]} \|G(t)\| < \infty$. For arbitrarily given $\varepsilon > 0$, since $\mathcal{G}$ is uniformly integrable, one can choose $N > 0$ such that

$$2\kappa \int_0^t \mathbf{1}_{\{s:\, \|f(s)\|_H > N\}} \|f(s)\|_H ds < \frac{\varepsilon}{2} \quad \text{for all} \quad f \in \mathcal{G}.$$

Since the retarded Green operator $G(t)$ is compact for $t > 0$ from (i), it thus follows that

$$\|G(t+\delta-s) - G(t-s)\| \to 0 \ \text{ for any } \ t-s > 0, \ \text{ as } \ \delta \to 0.$$

By the well-known Dominated Convergence Theorem, we have that

$$\lim_{\delta\to 0} \int_0^t \|G(t+\delta-s) - G(t-s)\| ds = 0.$$

Hence, for the above constant $N > 0$, there exists $\delta > 0$ such that

$$N \int_0^t \|G(t+\delta-s) - G(t-s)\| ds \le \frac{\varepsilon}{2} \quad \text{for all} \quad t \in [0, T).$$

Therefore, for such a $\delta > 0$ and all $f \in \mathcal{G}$, $t \in [0, T)$,

$$
\begin{aligned}
J_1(\delta, t, f) = & \int_0^t \mathbf{1}_{\{s:\, \|f(s)\|_H > N\}} \|G(t+u-s) - G(t-s)\| \|f(s)\|_H ds \\
& + \int_0^t \mathbf{1}_{\{s:\, \|f(s)\|_H \le N\}} \|G(t+\delta-s) - G(t-s)\| \|f(s)\|_H ds \\
& \le 2\kappa \int_0^t \mathbf{1}_{\{s:\, \|f(s)\|_H > N\}} \|f(s)\|_H ds + N \int_0^t \|G(t+\delta-s) - G(t-s)\| ds \\
& \le \varepsilon.
\end{aligned}
\tag{3.15}
$$

For the item $J_2(\delta, t, f)$, we have by virtue of the uniform integrability of $\mathcal{G}$ that

$$
\begin{aligned}
\lim_{\delta \to 0} \sup_{f \in \mathcal{G}} J_2(\delta, t, f) & \le \kappa \lim_{\delta \to 0} \sup_{f \in \mathcal{G}} \int_t^{t+\delta} \|f(s)\|_H ds \\
& = 0.
\end{aligned}
$$

Hence, the claim (ii) is shown and the whole proof is complete. □

4 LDP of Systems Driven by Lévy Processes

In this section, we are concerned with the strong solution of the following stochastic evolution equation driven by a Lévy noise,

$$
\begin{cases}
y(t) = \phi_0 + \displaystyle\int_0^t Ay(s)ds + \int_0^t \int_{-r}^0 d\eta(\theta) y(s+\theta) ds + bt + W(t) \\
\qquad\qquad\qquad\qquad\qquad + \displaystyle\int_0^t \int_X J(x) \tilde{N}(ds, dx), \quad t \in [0, T], \\
y(0) = \phi_0, \; y(t) = \phi_1(t), \quad t \in [-r, 0], \; \Phi = (\phi_0, \phi_1) \in \mathcal{V},
\end{cases}
\tag{4.1}
$$

where $b \in H$ and $W(t)$, $t \ge 0$, is an H-valued Q-Wiener process with $\operatorname{Tr} Q < \infty$. In the sequel, we impose the following exponential integrability condition on $J(\cdot)$:

$$
\int_X \|J(x)\|_H^2 \exp(c\|J(x)\|_H) \nu(dx) < \infty \quad \text{for all number} \quad c > 0. \tag{4.2}
$$

By virtue of a similar theory presented as in [1], it may be shown that there exists a unique strong solution to the equation (4.1). Moreover, for any $T \ge 0$ and almost all $\omega \in \Omega$,

$$
y(\cdot, \omega) \in D([0, T]; H) \cap L^2([0, T]; V).
$$

Now suppose that there exists a complete orthonormal system $\{e_n\}_{n=1}^\infty \subset V$ of H and a bounded sequence of nonnegative real numbers λ_k such that

$$
Qe_k = \lambda_k e_k, \qquad k = 1, 2, \cdots, \quad \text{and} \quad \sum_{k=1}^\infty \lambda_k < \infty. \tag{4.3}
$$

For any $m \in \mathbb{N}$, let $P_m : H \to H$ be the projection operator

$$P_m x = \sum_{k=1}^{m} \langle x, e_k \rangle_H e_k \in V, \quad x \in H, \tag{4.4}$$

and we introduce a mapping $y_m(t) = y_m(t, \cdot)$ from $D([0,T]; H)$ into $D([-r,T]; H)$ as follows: for $f \in D([0,T]; H)$, $y_m(t,f)$, $t \in [-r,T]$, is the unique strong solution of the equation

$$\begin{cases} y_m(t,f) = P_m\phi_0 + \int_0^t Ay_m(s,f)ds + \int_0^t \int_{-r}^0 d\eta(\theta) y_m(s+\theta, f)ds + P_m f(t), \quad t \in [0,T], \\ y_m(0,f) = P_m\phi_0, \;\; y_m(t,f) = P_m\phi_1(t), \quad t \in [-r,0], \quad \Phi = (\phi_0, \phi_1) \in \mathcal{V}. \end{cases} \tag{4.5}$$

For any $n \geq 1$, let

$$L^n(t) = bt + \frac{1}{\sqrt{n}} W(t) + \frac{1}{n} \int_0^t \int_X J(x) \tilde{N}_n(ds, dx).$$

Then it is easy to see that $y^{m,n}(t) := y_m(t, L^n(t))$ is the unique solution of the following stochastic differential equation:

$$\begin{cases} y^{m,n}(t) = P_m\phi_0 + \int_0^t Ay^{m,n}(s)ds + \int_0^t \int_{-r}^0 d\eta(\theta) y^{m,n}(s+\theta)ds + b_m t + \frac{1}{\sqrt{n}} W_m(t) \\ \qquad\qquad + \frac{1}{n} \int_0^t \int_X J_m(x) \tilde{N}_n(ds,dx), \quad t \in [0,T], \\ y^{m,n}(0) = P_m\phi_0, \;\; y^{m,n}(t) = P_m\phi_1(t), \quad t \in [-r,0], \quad \Phi = (\phi_0,\phi_1) \in \mathcal{V}, \end{cases} \tag{4.6}$$

where $J_m(x) = P_m J(x) = \sum_{k=1}^m \langle J(x), e_k \rangle_H e_k$, $W_m(t) = P_m W(t)$ and $b_m = P_m b$ for $m \in \mathbb{N}$.

Lemma 4.1. *For arbitrary $T \geq 0$, and $\delta > 0$, it holds true that*

$$\lim_{m\to\infty} \limsup_{n\to\infty} \frac{1}{n} \log \mathbb{P}\Big(\sup_{0 \leq t \leq T} \|y^{m,n}(t) - y^n(t)\|_H > \delta \Big) = -\infty, \tag{4.7}$$

where $y^{m,n}(\cdot)$ and $y^n(\cdot)$ are the strong solutions given in (4.6) and (1.12), respectively.

Proof. For any m, $n \in \mathbb{N}$ and $t \in [0,T]$, let

$$x^{m,n}(t) = ne^{-(\lambda+M)t}(y^{m,n}(t) - y^n(t)). \tag{4.8}$$

It is easy to see that

$$\begin{aligned} x^{m,n}(t) = &\int_0^t Ax^{m,n}(s)ds + \int_0^t \int_{-r}^0 d\eta(\theta) x^{m,n}(s+\theta)ds + \int_0^t \int_X (J_m(x) - J(x)) \tilde{N}_n(ds,dx) \\ &- (\lambda + M) \int_0^t x^{m,n}(s)ds + (P_m\phi_0 - \phi_0) + n(b_m - b)t + \sqrt{n}(W_m(t) - W(t)). \end{aligned} \tag{4.9}$$

For any fixed $\gamma > 0$, let $g(y) = (1+\gamma\|y\|_H^2)^{1/2}$, $y \in H$. Then it is easy to see that

$$g'(y) = \gamma(1+\gamma\|y\|_H^2)^{-1/2}y, \qquad y \in H,$$
$$g''(y) = -\gamma^2(1+\gamma\|y\|_H^2)^{-3/2}y\otimes y + \gamma(1+\gamma\|y\|_H^2)^{-1/2}I_H, \qquad y \in H, \tag{4.10}$$

where I_H stands for the identity operator on H and $y \otimes y$ is a linear operator on H defined by: $(y\otimes y)x = \langle y, x\rangle_H y$, $x \in H$. It is immediate to see the following relations

$$\sup_{y\in H}\|g''(y)\| \le \gamma, \qquad \sup_{y\in H}\|g'(y)\|_H \le \gamma^{1/2}. \tag{4.11}$$

Let $q(y) = \exp(g(y))$, $y \in H$. By Taylor's expansion, for $m \in \mathbb{N}$, there exists a $\theta_m \in (0,1)$ such that

$$\begin{aligned}
&\exp\big[g(y+J_m(x)-J(x)) - g(y)\big] - 1 - \langle g'(y), J_m(x)-J(x)\rangle_H \\
&\quad= e^{-g(y)}\Big[q(y+J_m(x)-J(x)) - q(y) - q(y)\langle g'(y), J_m(x)-J(x)\rangle_H\Big] \\
&\quad= \frac{1}{2}e^{-g(y)}\langle q''(y+\theta_m(J_m(x)-J(x))), (J_m(x)-J(x))\otimes(J_m(x)-J(x))\rangle_H, \qquad x \in X.
\end{aligned} \tag{4.12}$$

Note that

$$q''(y) = q(y)g'(y)\otimes g'(y) + q(y)g''(y),$$

which, together with (4.11), immediately yields that

$$\|q''(y)\| \le \gamma q(y) \quad \text{for all} \quad y \in H. \tag{4.13}$$

Hence, by virtue of (4.12) and (4.13), it follows for some $\tilde{\theta}_m \in (0,\theta_m)$ that

$$\begin{aligned}
&\Big|\exp\big[g(y+J_m(x)-J(x)) - g(y)\big] - 1 - \langle g'(y), J_m(x)-J(x)\rangle_H\Big| \\
&\qquad\le \gamma\exp[g(y+\theta_m(J_m(x)-J(x))) - g(y)]\|J_m(x)-J(x)\|_H^2 \\
&\qquad= \gamma\exp[\langle g'(y+\tilde{\theta}_m(J_m(x)-J(x))), \theta_m(J_m(x)-J(x))\rangle_H]\|J_m(x)-J(x)\|_H^2 \\
&\qquad\le \gamma\exp[\gamma^{1/2}\|J_m(x)-J(x)\|_H]\|J_m(x)-J(x)\|_H^2.
\end{aligned} \tag{4.14}$$

For any $s \in [0,T]$, let us put

$$\begin{aligned}
h(x^{m,n}(s)) := {}& \langle g'(x^{m,n}(s)), Ax^{m,n}(s)\rangle_{V,V^*} + \Big\langle g'(x^{m,n}(s)), \int_{-r}^0 d\eta(\theta)x^{m,n}(s+\theta)\Big\rangle_H \\
&- (\lambda+M)\langle g'(x^{m,n}(s)), x^{m,n}(s)\rangle_H \\
&+ n\int_X\Big(\exp\big[g(x^{m,n}(s)+J_m(x)-J(x)) - g(x^{m,n}(s))\big] - 1 \\
&- \langle g'(x^{m,n}(s)), J_m(x)-J(x)\rangle_H\Big)\nu(dx) + n\langle b^m - b, g'(x^{m,n}(s))\rangle_H \\
&+ n\sum_{k=m+1}^{\infty}\lambda_k\langle g'(x^{m,n}(s))\otimes g'(x^{m,n}(s)) + g''(x^{m,n}(s))e_k, e_k\rangle_H \\
&+ \langle P_m\phi_0 - \phi_0, g'(x^{m,n}(s))\rangle_H.
\end{aligned} \tag{4.15}$$

Note that by virtue of (3.1), (3.2) and (1.4) and (4.10), we have, for any $m,\ n \in \mathbb{N}$ and $t \geq 0$,

$$
\begin{aligned}
&\int_0^t \langle g'(x^{m,n}(s)), Ax^{m,n}(s)\rangle_{V,V^*} ds + \int_0^t \Big\langle g'(x^{m,n}(s)), \int_{-r}^0 d\eta(\theta) x^{m,n}(s+\theta)\Big\rangle_H ds \\
&\qquad - (\lambda + M)\int_0^t \langle g'(x^{m,n}(s)), x^{m,n}(s)\rangle ds \\
&\leq \int_0^t \Big[\gamma(1+\gamma\|x^{m,n}(s)\|_H^2)^{-\frac{1}{2}}\Big]\Big(\langle x^{m,n}(s), Ax^{m,n}(s)\rangle_{V,V^*} \\
&\qquad + \Big\langle x^{m,n}(s), \int_{-r}^0 d\eta(\theta)x^{m,n}(s+\theta)\Big\rangle_H\Big) ds \\
&\qquad - (\lambda+M)\int_0^t \Big[\gamma(1+\gamma\|x^{m,n}(s)\|_H^2)^{-1/2}\Big]\|x^{m,n}(s)\|_H^2 ds \\
&\leq \Big(-\frac{\alpha}{\beta} + M + \lambda\Big)\int_0^t \Big[\gamma(1+\gamma\|x^{m,n}(s)\|_H^2)^{-1/2}\Big]\|x^{m,n}(s)\|_H^2 ds \\
&\qquad - (\lambda+M)\int_0^t \Big[\gamma(1+\gamma\|x^{m,n}(s)\|_H^2)^{-1/2}\Big]\|x^{m,n}(s)\|_H^2 ds \\
&\leq 0,
\end{aligned}
\tag{4.16}
$$

which, in addition to (4.15), immediately yields that for $t \in [0, T]$,

$$
\int_0^t h(x^{m,n}(s))ds \leq TC_{\gamma,m} n \tag{4.17}
$$

where

$$
\begin{aligned}
C_{\gamma,m} = \gamma \int_X &\|J_m(x) - J(x)\|_H^2 \exp\big[\gamma^{1/2}\|J_m(x) - J(x)\|_H\big]\nu(dx) \\
&+ \gamma^{1/2}\|b_m - b\|_H + \gamma^{1/2}\|P_m\phi_0 - \phi_0\|_H + 2\gamma\sum_{k=m+1}^{\infty}\lambda_k.
\end{aligned}
\tag{4.18}
$$

In view of (4.2), it is easy to see that $C_{\gamma,m} < \infty$ for each $m \in \mathbb{N}$, and so for $t \in [0, T]$,

$$
\int_0^t h(x^{m,n}(s))ds < \infty \quad \text{for each} \quad m,\ n \in \mathbb{N}.
$$

Now applying Itô's formula to $\exp(g(x^{m,n}(t))$ first and then to

$$
\exp\Big(g(x^{m,n}(t)) - g(\phi_0) - \int_0^t h(x^{m,n}(s))ds\Big), \qquad t \in [0, T],
$$

we may immediately get that

$$
M_g^{m,n}(t) := \exp\Big(g(x^{m,n}(t)) - g(\phi_0) - \int_0^t h(x^{m,n}(s))ds\Big), \qquad t \in [0, T],
$$

is an $\mathscr{F}_t$-local martingale. Hence, for arbitrary $\delta > 0$ and $m, n \in \mathbb{N}$, by setting $\delta_1 = e^{-(\lambda+M)T}\delta > 0$, we have in view of (4.15) and (4.17) that

$$\begin{aligned}
&\mathbb{P}\Big\{\sup_{0\le t\le T}\|y^{m,n}(t)-y^n(t)\|_H>\delta\Big\}\\
&=\mathbb{P}\Big\{\sup_{0\le t\le T}\|x^{m,n}(t)\|_H>n\delta_1\Big\}\\
&\le\mathbb{P}\Big\{\sup_{0\le t\le T}g(x^{m,n}(t))\ge(1+\gamma(n\delta_1)^2)^{1/2}\Big\}\\
&=\mathbb{P}\Big\{\sup_{0\le t\le T}\Big(g(x^{m,n}(t))-g(\phi_0)-\int_0^t h(x^{m,n}(s))ds\\
&\qquad\qquad\qquad+g(\phi_0)+\int_0^t h(x^{m,n}(s))ds\Big)\ge(1+\gamma(n\delta_1)^2)^{1/2}\Big\}\\
&\le\mathbb{P}\Big\{\sup_{0\le t\le T}\Big(g(x^{m,n}(t))-g(\phi_0)-\int_0^t h(x^{m,n}(s))ds\Big)\ge(1+\gamma(n\delta_1)^2)^{\frac12}-g(\phi_0)-TC_{\gamma,m}n\Big\}\\
&\le\mathbb{E}\Big[\sup_{0\le t\le T}M_g^{m,n}(t)\Big]\exp\Big(-(1+\gamma(n\delta_1)^2)^{1/2}+g(\phi_0)+TC_{\gamma,m}n\Big).
\end{aligned}\tag{4.19}$$

Since $M_g^{m,n}(t)$ is a non-negative local martingale, it is a supermartingale and thus there is

$$\mathbb{E}\Big[\sup_{0\le t\le T}M_g^{m,n}(t)\Big]\le 1.\tag{4.20}$$

Hence, both (4.19) and (4.20) imply that for any $m\in\mathbb{N}$,

$$\begin{aligned}
&\limsup_{n\to\infty}\frac{1}{n}\log\mathbb{P}\Big(\sup_{0\le t\le T}\|y^{m,n}(t)-y^n(t)\|_H>\delta\Big)\\
&\qquad\le\limsup_{n\to\infty}\frac{1}{n}\Big[-(1+\gamma(n\delta_1)^2)^{1/2}+g(\phi_0)+TC_{\gamma,m}n\Big]\\
&\qquad\le-\gamma\delta_1+TC_{\gamma,m}.
\end{aligned}\tag{4.21}$$

For any fixed $\gamma > 0$, it is easy to see by Dominated Convergence Theorem that

$$\lim_{m\to\infty}C_{\gamma,m}=0.$$

Thus, letting $m\to\infty$ in (4.21), we get

$$\lim_{m\to\infty}\limsup_{n\to\infty}\frac{1}{n}\log\mathbb{P}\Big(\sup_{0\le t\le T}\|y^{m,n}(t)-y^n(t)\|_H>\delta\Big)\le-\gamma\delta_1$$

which, letting $\gamma\to\infty$ further, implies immediately the desired result (4.7). The proof is thus complete. □

For arbitrary $y\in H$, we put

$$H(y)=\int_X\Big[\exp(\langle J(x),y\rangle_H)-1-\langle J(x),y\rangle_H\Big]\nu(dx)+\langle Qy,y\rangle_H+\langle b,y\rangle_H.$$

Also, for any $u \in H$, we define

$$J(u) = \sup_{y \in H}[2\langle u, y\rangle_H - H(y)]. \tag{4.22}$$

For arbitrarily given $f \in D([0,T];H)$, let $y(f) = y(\cdot, f)$ be the unique solution to the following equation: for $t \in [0,T]$,

$$\begin{cases} y(t,f) = \phi_0 + \int_0^t Ay(s,f)ds + \int_0^t A_1 y(s-r,f)ds + \int_0^t \int_{-r}^0 A_0(\theta)y(s+\theta,f)d\theta ds + f(t), \\ y(0,f) = \phi_0 \in H, \;\; y(t,f) = \phi_1(t) \in L^2([-r,0];V), \qquad t \in [-r,0], \end{cases} \tag{4.23}$$

where $r > 0$, $A_0(\cdot) \in L^2([-r,0];\mathscr{L}(H))$ and $A_1 \in \mathscr{L}(H)$.

Lemma 4.2. *Assume that A generates a compact C_0-semigroup e^{tA}, $t \geq 0$, and there exist sequences $\{\alpha_k\}$, $\{\beta_{1k}\} \in \mathbb{R}^1$ and $\beta_{0k} \in L^2([-r,0];\mathbb{R}^1)$, $k \in \mathbb{N}$, such that*

$$Ae_k = \alpha_k e_k, \quad A_1 e_k = \beta_{1k} e_k, \quad A_0(\theta)e_k = \beta_{0k}(\theta)e_k, \quad k \in \mathbb{N}, \quad \theta \in [-r,0].$$

where $\{e_k\} \subset V$ is the complete orthonormal basis of H given in (4.3). Then for any $T \geq 0$ and $\delta > 0$, it holds that

$$\lim_{m\to\infty} \sup_{\{f \in D([0,T];H):\, \int_0^T J(f(s))ds \leq \delta\}} \sup_{0\leq t\leq T} \|y_m(t,f) - y(t,f)\|_H = 0,$$

where $y_m(t,f) = P_m y(t,f) = \sum_{k=1}^m \langle y(t,f), e_k\rangle_H e_k$ is given in (4.5).

Proof. Recall that the retarded Green operator $G(t)$, $t \in \mathbb{R}^1$, is the unique solution of the equation

$$G(t) = \begin{cases} e^{tA} + \int_0^t e^{(t-s)A} A_1 G(s-r)ds + \int_0^t e^{(t-s)A} \int_{-r}^0 A_0(\theta)G(s+\theta)d\theta ds, & t \geq 0, \\ \mathrm{O}, & t < 0. \end{cases} \tag{4.24}$$

By assumption, for any $m \in \mathbb{N}$ the projection operator P_m commutes with the C_0-semigroup e^{tA}, $t \geq 0$, and operators A_1, $A_0(\cdot)$, a fact which implies that for any $m \in \mathbb{N}$, the projection operator P_m commutes with $G(t)$, $t \in \mathbb{R}^1$. For any $f \in D([0,T];H)$ with $\int_0^T J(f(s))ds < \infty$, the solution of the equation (4.23) is represented in terms of $G(t)$, $t \in \mathbb{R}^1$, by

$$\begin{aligned} y(t,f) = {} & G(t)\phi_0 + \int_{-r}^0 G(t-\theta-r)A_1\phi_1(\theta)d\theta + \int_{-r}^0\int_{-r}^\theta G(t-\theta+\tau)A_0(\tau)\phi_1(\theta)d\tau d\theta \\ & + \int_0^t G(t-s)f(s)ds, \quad t \in [0,T], \end{aligned} \tag{4.25}$$

which immediately implies that

$$y_m(t,f) = P_m(y(t,f)) \quad \text{for any} \quad t \in [0,T], \quad m \in \mathbb{N}.$$

By using Theorem 3.1, [3] with a slightly different modification, we obtain that the set $\{f \in D([0,T];H) : \int_0^T J(f(s))ds \leq \delta\}$, $\delta > 0$, is uniformly integrable on the finite measure space $([0,T];\mathscr{B}([0,T]),L)$ where L stands for the standard Lebesgue measure. Based on this fact, it follows further from Proposition 3.1 that $\mathcal{S}_T = \{y(f) : \int_0^T J(f(s))ds \leq \delta\}$ is relatively compact in $C([0,T];H)$. Therefore, for any $\varepsilon > 0$, there exist $f_1, f_2, \cdots, f_N \in \{f \in D([0,T];H) : \int_0^T J(f(s))ds \leq \delta\}$ such that

$$\mathcal{S}_T \subset \bigcup_{k=1}^N B(y(f_k), \varepsilon/3),$$

where $B(y(f_k), \varepsilon/3)$ is the ball centered at $y(f_k)$ with radius $\varepsilon/3$ in $C([0,T];H)$. Since

$$\lim_{m\to\infty} \|y_m(t,f_k) - y(t,f_k)\|_H = 0 \quad \text{for each} \quad k \in \mathbb{N},$$

there exists $M \geq 1$ such that

$$\sup_{0\leq t\leq T} \|y_m(t,f_k) - y(t,f_k)\|_H \leq \frac{\varepsilon}{3} \quad \text{for all} \quad k \leq N,\ \ m \geq M.$$

Therefore, for any $f \in D([0,T];H)$ with $\int_0^T J(f(s))ds \leq \delta$ and $\delta > 0$, there is $k \leq N$ such that $y(f) \in B(y(f_k), \varepsilon/3)$, and if $m \geq M$, it further follows that

$$\begin{aligned}
\sup_{0\leq t\leq T} \|y_m(t,f) - y(t,f)\|_H &\leq \sup_{0\leq t\leq T} \|y_m(t,f) - y_m(t,f_k)\|_H + \sup_{0\leq t\leq T} \|y_m(t,f_k) - y(t,f_k)\|_H \\
&\quad + \sup_{0\leq t\leq T} \|y(t,f_k) - y(t,f)\|_H \\
&\leq 2 \sup_{0\leq t\leq T} \|y(t,f_k) - y(t,f)\|_H + \sup_{0\leq t\leq T} \|y_m(t,f_k) - y(t,f_k)\|_H \\
&\leq \frac{2\varepsilon}{3} + \frac{\varepsilon}{3} \\
&= \varepsilon.
\end{aligned} \tag{4.26}$$

The proof is thus complete. □

Now we are in a position to state the main results of this work.

Theorem 4.1. *Under the same conditions as in Lemma 4.2, the law* $\mu_n(\cdot)$ *of* $y^n(\cdot)$, $t \in [0,T]$, *in (1.12) satisfies a LDP on* $L^2([0,T];H)$, $T \geq 0$, *with the rate functional* I *given by: for* $z \in L^2([0,T];H)$,

$$I(z) = \begin{cases} \inf\Big\{\dfrac{1}{2}\displaystyle\int_0^T J(u(s))ds : \ u \in L^2([0,T];H) \text{ such that } J(u) \in L^1([0,T];H) \text{ and} \\ \qquad\qquad \displaystyle\int_{-r}^0\int_{-r}^\theta G(t-\theta+\tau)d\eta(\tau)\phi_1(\theta)d\theta \\ \qquad\qquad +G(t)\phi_0 + \displaystyle\int_0^t G(t-s)u(s)ds = z(t),\ t \in [0,T]\Big\}, \\ \infty, \qquad \text{otherwise.} \end{cases}$$

Proof. Let $\nu_n, n \geq 1$, be the law of the Lévy process $\{L^n(t), t \in [0,T]\}$. It is known by [4] that $\{\nu_n,\, n \geq 1\}$ satisfies a LDP with the rate function: for $z \in L^2([0,T];H)$,

$$I(z) = \begin{cases} \inf\Big\{\dfrac{1}{2}\displaystyle\int_0^T J(u(s))ds:\ u \in L^2([0,T];H) \text{ such that } J(u) \in L^1([0,T];H) \text{ and} \\ \qquad\qquad\qquad\qquad \displaystyle\int_0^t G(t-s)u(s)ds = z(t),\, t \in [0,T]\Big\}, \\ \infty, \qquad \text{otherwise.} \end{cases}$$

By applying the well-known contraction principle (cf. [23]), we see that $\{y^{m,n}\}$ satisfies a LDP on $L^2([0,T];H)$ with a rate functional I_m, $m \in \mathbb{N}$, given as follows: for $z \in L^2([0,T];H)$,

$$I_m(z) = \begin{cases} \inf\Big\{\dfrac{1}{2}\displaystyle\int_0^T J(u(s))ds:\ u \in L^2([0,T];H) \text{ such that } J(u) \in L^1([0,T];H) \text{ and} \\ \qquad\qquad \displaystyle\int_{-r}^0\int_{-r}^\theta G(t-\theta+\tau)d\eta(\tau)P_m\phi_1(\theta)d\theta \\ \qquad\qquad + G(t)P_m\phi_0 + \displaystyle\int_0^t G(t-s)P_m u(s)ds = z(t),\, t \in [0,T]\Big\}, \\ \infty, \qquad \text{otherwise.} \end{cases}$$

According to the generalized contraction principle, Th. 4.2, in [8] and Lemmas 4.1 and 4.2, the desired result follows now. The proof is thus complete. □

Corollary 4.1. *Assume that A generates a compact C_0-semigroup e^{tA}, $t \geq 0$, such that*

$$Ae_k = \alpha_k e_k, \quad \alpha_k \in \mathbb{R}^1, \quad k \in \mathbb{N},$$

where $\{e_k\} \subset V$ is the complete orthonormal basis of H given in (4.3). Suppose further that $A_0(\cdot) = a_0(\cdot)I_H$, $a_0(\cdot) \in L^2([-r,0];\mathbb{R}^1)$, $A_1 = a_1 I_H$, $a_1 \in \mathbb{R}^1$, in (1.2). Then the law $\mu_n(\cdot)$ of $y^n(\cdot)$, $t \in [0,T]$, in (1.12) satisfies a LDP on $L^2([0,T];H)$, $T \geq 0$, with the rate functional I given by: for $z \in L^2([0,T];H)$,

$$I(z) = \begin{cases} \inf\Big\{\dfrac{1}{2}\displaystyle\int_0^T J(u(s))ds:\ u \in L^2([0,T];H) \textit{ such that } J(u) \in L^1([0,T];H) \textit{ and} \\ \qquad\qquad \displaystyle\int_{-r}^0\int_{-r}^\theta G(t-\theta+\tau)d\eta(\tau)\phi_1(\theta)d\theta \\ \qquad\qquad + G(t)\phi_0 + \displaystyle\int_0^t G(t-s)u(s)ds = z(t),\, t \in [0,T]\Big\}, \\ \infty, \qquad \textit{otherwise.} \end{cases}$$

Remark 4.1. In the work of [20], a LDP is established for infinite dimensional Ornstein-Uhlenbeck processes driven by Lévy noise under the assumption that $\lambda = 0$ in (3.2). By using the transform (4.8) and carrying out a similar argument as in Lemma 4.1, we may actually see that this restriction could be removed.

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