Brain Wave Solution to the Modified Schrodinger Equation

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Abstract: What is your perception of reality? Is it same as my perception of reality? What defines perception? Does perception like ice cream? And at last but not least, does perception even exist? Questions like these are the basis of the modern doctrine of consciousness. After a long time of research we have not been able to understand much about consciousness, however we humans have been able to find a fundamental property of consciousness without even fully understanding the 'consciousness' itself. The fundamental property of consciousness is the memory.
In Bohmian mechanics (Bohm's interpretation), the pilot wave acts as a navigator for a particle, and so happens in human brain. In this paper we contemplate the bohm's interpretation and its indirect relation with consciousness, by showing its relation with human memory. The bohm's interpretation is an interpretation of modified Schrodinger equation, which itself is connected with the memory of quantum system.

Keywords: Quantum Mechanics, Bohmian Mechanics, Pilot Wave, Consciousness, Thermal Memory, Physics & Brain, Nature of Reality, Schrodinger Equation, Modified Schrodinger Equation, de Broglie and Bohm Interpretation.

1. INTRODUCTION

The Bohm's interpretation, with a system of N particles is described by the wave function and the configuration q_k, or we can say by the actual position of the quantum objects. This implies that bohm has to add an "equation of navigation" or "equation of motion" for the positions to the formalism. Let's assume a wave function $\Psi = R\exp(iS/\hbar)$, the navigation equation for the position q of a spin-less particle in 1-particle case takes the form:

$$\frac{dq}{dt} = \frac{\vec{\nabla S}}{m} \tag{1}$$

This equation is dependent on the initial configuration to fix the motion uniquely. The generalization of >1 particle case including spin is straightforward[1]. Using 'Born rule' i.e. $\rho = |\psi|^2$ to give initial conditions for the position, the equation of continuity

$$\frac{d\rho}{dt} + \nabla j = 0 \tag{2}$$

Using the usual quantum mechanical probability current

$$j = \frac{\hbar}{2mi}[\Psi^*(\nabla\Psi) - (\nabla\Psi^*)\Psi \tag{3}$$

$$= \rho\frac{\nabla S}{m} \tag{4}$$

ensures that the position remains $|\Psi|^2$ distributed. We do this to ensure that in terms of position any measurement yields exactly the result of standard formalism, so that the navigator equation is consistent with requirements of quantum mechanics.
In ordinary Quantum mechanics the probability current refers to the probability to measure a certain position. In Bohmian mechanics it is viewed as the probability of the particle to be at a certain position; independent of any measurement.

The work on this doctrine was originally did by Louis de Broglie in 1920 [2] i.e. why it is also called "de Broglie Bohm" theory (interpretation). Which [1] presents a version called "Bohmian Mechanics", the books by Bohm and Hiley [3] are more closer to the orginal research of bohm from1952 [4]—also called ontological interpretation of Quantum mechanics.

2. Brain wave activity as solution to the Modified Schrodinger equation

To start, first we derive Modified Schrodinger Equation by contemplating the thermal phenomena. The thermal history of the system (universe) can be described by the generalized Fourier equation [5] [6]

$$q(t) = -\int_{-\infty}^{t} K(t-t')\nabla T(t')dt' \tag{5}$$

Where,

$$\text{K(t-t}')=\{\textit{thermal history (thermal memory)}$$

$$\nabla\text{T(t}')\text{dt}'=\{\textit{diffusion}$$

In Eq. (5) q(t) is the density of the energy flux, T is the temperature of the system.

$$K(t-t') = \frac{K}{\tau}\exp\left[-\frac{(t-t')}{\tau}\right], \tag{6}$$

Where τ denotes relaxation time,

$$K(t-t') = \begin{cases} K\delta(t-t') & \text{(diffusion)} \\ K = constant & \text{(wave)} \\ \dfrac{K}{\tau}\exp\left[-\dfrac{t-t')}{\tau}\right] & \text{(hyperbolic diffusion)} \end{cases}$$

The hyperbolic diffusion rewritten, becomes

$$\frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau}\frac{\partial T}{\partial t} = \frac{D_T}{\tau}\nabla^2 T \tag{7}$$

For $\tau \to 0$, the Eq. (7) becomes the Fourier thermal equation

$$\frac{\partial D}{\partial t} = D_T \nabla^2 T \tag{8}$$

and D_T is thermal diffusion coefficient. The systems with very short τ have very short thermal history. For $\tau\to\infty$ Eq. (7) has the form of *ballistic thermal equation.* The ballistic phonons or electrons are those for which $\tau\to\infty$. The experiments related with ballistic phonons or electrons demonstrate the existence of the wave motion on the lattice scale or on the electron gas scale.

$$\frac{\partial^2 T}{\partial t^2} = \frac{D_T}{\tau}\nabla^2 T \tag{9}$$

For the systems with very long thermal history Eq. (7) is time symmetric with of course no arrow of time. For Eq. (9) shape does not change when t→ -t.

In Eq. (7) we define:

$$\nu = \left(\frac{D_T}{\tau}\right), \tag{10}$$

Velocity of thermal wave propagation, and

$$\lambda = \nu\tau \tag{11}$$

Where λ is defines the mean free path of heat carriers. With formula (10), the Eq. (7) can be rewritten as:

$$\frac{1}{\nu^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau\nu^2}\frac{\partial T}{\partial t} = \nabla^2 T \tag{12}$$

The equation

$$\frac{1}{\nu^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{D}\frac{\partial T}{\partial t} = \nabla^2 T$$

is the hyperbolic partial differential equation on one hand, and Fourier equation on the other hand

$$\frac{1}{D}\frac{\partial T}{\partial t} = \nabla^2 T \tag{13}$$

and Schrodinger equation

$$i\hbar\frac{\partial \Psi}{\partial \mathrm{t}} = \nabla^2 \Psi \tag{14}$$

are both parabolic equations. Using substitution

$$t \leftrightarrow it, \quad \Psi \leftrightarrow T \tag{15}$$

Fourier Eq. (13) can be rewritten as:

$$i\hbar\frac{\partial \Psi}{\partial t} = -D\hbar\nabla^2\Psi \tag{16}$$

Using comparison with Schrodinger equation, we get:

$$D_T\hbar = \frac{\hbar^2}{2m} \tag{17}$$

$$\Longrightarrow \quad D_T = \frac{\hbar}{2m} \tag{18}$$

If we consider $D_T = \tau\nu^2$ in Eq. (10), we obtain from Eq. (18)

$$\tau = \frac{\hbar}{2m\nu_{\hbar}^{2}} \tag{19}$$

The formula (19) describes the τ for the quantum thermal processes.

Starting with Schrodinger equation for a particle with mass m in potential V:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi \tag{20}$$

Substituting (15), we get:

$$\hbar\frac{\partial T}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\mathrm{T} - \mathrm{VT} \tag{21}$$

$$\Longrightarrow \quad \frac{\partial T}{\partial t} = \frac{\hbar}{2m}\nabla^2 T - \frac{V}{\hbar}T \tag{22}$$

In Eq. (22) $\tau = 0$, we get:

$$\tau\frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} + \frac{V}{\hbar}T = \frac{\hbar}{2m}\nabla^2 T \tag{23}$$

and for $\tau \neq 0$, we get:

$$\tau = \frac{\hbar}{2m\nu^2}$$

We take relaxation time τ as real constant in Eq. (23), we get:

$$\frac{1}{\nu^2}\frac{\partial^2 T}{\partial t^2} + \frac{2m}{\hbar}\frac{\partial T}{\partial t} + \frac{2Vm}{\hbar^2}T = \nabla^2 T \tag{24}$$

With substitution of Eq. (11) in Eq. (23), we get:

$$i\hbar\frac{\partial\Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\nabla^2\Psi - \tau\hbar\frac{\partial^2\Psi}{\partial t^2} \tag{24}$$

The new relaxation term

$$\tau\hbar\frac{\partial^2\Psi}{\partial t^2} \tag{25}$$

describes the interaction of the particle with mass m with space-time.

The relaxation time τ can be calculated as:

$$\tau^{-1} = (\tau_{e-p}^{-1} + \cdots + \tau_{Planck}^{-1} \tag{26}$$

Where $\tau_{e\text{-}p}$ denotes the scattering of the particle m on the electron-positron pair ($\tau_{e\text{-}p}\sim 10^{-17}$S). The shortest relaxation time τ_{Planck} is the Planck time ($\tau_{Planck}\sim 10^{-43}$S).

From Eq.(26) we conclude that $\tau\approx\tau_{Planck}$, and Eq. (24) can be rewritten as:

$$i\hbar\frac{\partial\Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\nabla^2\Psi - \tau_{Planck}\hbar\frac{\partial^2\Psi}{\partial t^2} \tag{27}$$

Where,

$$\tau_{Planck} = \frac{1}{2}\left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = \frac{\hbar}{2M_p c^2} \tag{28}$$

In Eq. (28) M_p is the planck mass.

Using the value of Eq. (28) in Eq. (27), we get:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi - \frac{\hbar^2}{2M_p}\nabla^2\Psi + \frac{\hbar^2}{2M_p}\nabla^2\Psi - \frac{\hbar^2}{2M_p c^2}\frac{\partial^2\Psi}{\partial t^2} \tag{29}$$

Last two terms in the Eq. (29) are special because they can be defined as the *Bhomian Pilot wave*

$$\frac{\hbar^2}{2M_p}\nabla^2\Psi - \frac{\hbar^2}{2M_p c^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \tag{30}$$

$$\Longrightarrow\ \nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \tag{31}$$

We can observe that pilot wave Ψ does not depend on the mass of the particle.

Using the postulate from Eq. (31) in Eq. (29), we get:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi - \frac{\hbar^2}{2M_p}\nabla^2\Psi \tag{32}$$

$$\Longrightarrow \text{simultaneously}\ \left\{\frac{\hbar^2}{2M_p}\nabla^2\Psi - \frac{\hbar^2}{2M_p c^2}\frac{\partial^2\Psi}{\partial t^2} = 0\right. \tag{33}$$

We can write Eq. (24) in operator form i.e.

$$\hat{E} = \frac{\hat{p}}{2m} + \frac{1}{2M_pc^2}\hat{E}^2 \quad (34)$$

Where $\hat{E}$ and $\hat{p}$ are operators for energy and momentum of the particle with mass m.

- Eq. (34) is the new dispersion relation for quantum particle with mass m
- From Eq. (24) we conclude that Schrodinger quantum mechanics is valid for particles with $m \ll M_p$.
- Pilot wave exists independent to the mass of the particle.

For the particle with $m \ll M_p$ Eq, (32) takes the form of:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi \quad (35)$$

For the case $m \approx M_p$ Eq. (32) becomes,

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2M_p}\nabla^2\Psi + V\Psi \quad (36)$$

but, using the case $m \approx M_p$ in Eq. (33), we get:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2M_pc^2}\frac{\partial^2\Psi}{\partial t^2} + V\Psi \quad (37)$$

$$\Longrightarrow \quad \frac{\hbar^2}{2M_pc^2}\frac{\partial^2\Psi}{\partial t^2} + i\hbar\frac{\partial\Psi}{\partial t} - V\Psi = 0 \quad (38)$$

For Eq. (38) we look for the solution in the form of:

$$\Psi(x,t) = e^{-i\omega}u(x) \quad (39)$$

Substituting the (39) in Eq. (38), we get:

$$\frac{\hbar^2}{2M_pc^2}\omega^2 - \omega\hbar + V(x) = 0 \quad (40)$$

With the solution,

For $\frac{M_pc^2}{2} > V$,

$$\omega_1 = -\frac{M_p c^2 + M_p c^2 \sqrt{1 - \frac{2V}{M_p c^2}}}{\hbar}$$

(41)

$$\omega_2 = -\frac{M_p c^2 - M_p c^2 \sqrt{1 - \frac{2V}{M_p c^2}}}{\hbar}$$

For $\frac{M_p c^2}{2} < V,$

$$\omega_1 = -\frac{M_p c^2 + i M_p c^2 \sqrt{\frac{2V}{M_p c^2} - 1}}{\hbar}$$

(42)

$$\omega_2 = -\frac{M_p c^2 - i M_p c^2 \sqrt{\frac{2V}{M_p c^2} - 1}}{\hbar}$$

Both (41) and (42) describe the "string oscillation", formula (30) damped oscillation, and formula (31) over damped string oscillation.

For elementary particle physics, the internal energy $M_p c^2$ is the maximum energy per particle in the universe.
In that case we can argue that the solution (41) is the valid solution[7].

For $\frac{M_p c^2}{2} < V$, we get:

$$\omega_1 = \frac{2 M_p c^2}{\hbar}$$

(43)

$$\omega_2 = \frac{V}{\hbar}$$

The ω_1 represents Planck frequency $\omega_1 = \tau_p^{-1}$ and ω_2 is the frequency for Brain Waves.

Figure α represents ω_1 as the function of the ratio

$$\frac{2V}{M_p c^2} \tag{44}$$

We can observe from Fiqure α, for the potential energy $V \approx 10^{-15}$eV angular frequency of the brain waves is of the order 10Hz.

Considering that M_p equals to the mass of human neuron (10^5g) both Eqs. (30) and (31) describe the human neuron oscillations or emission of brain waves.

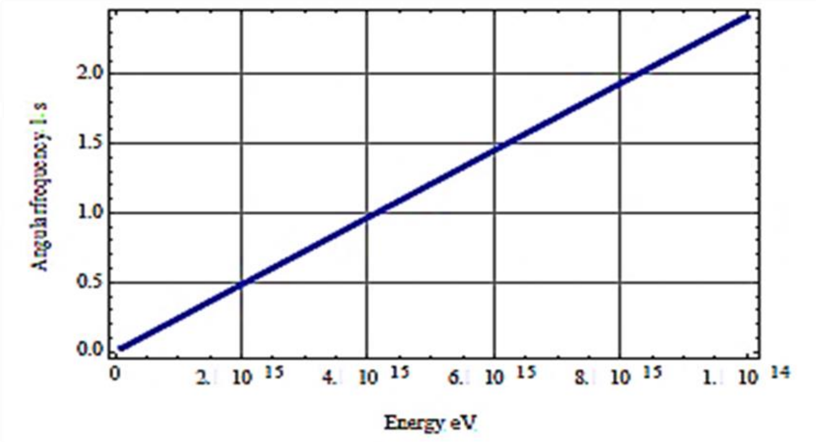

Figure α. Angular frequency as the function of energy.

3. CONCLUSION

In this paper we contemplated the relationship between ‘Brain waves’ and the ‘Bohmian Mechanics’, and we did it in order to imply an indirect relationship between the “Consciousness” and ‘Physics’. We have seen that the brain waves indeed do act as solutions to the modified Schrodinger equation, thus giving a link between the brain waves and Quantum mechanics. We know that consciousness has the fundamental property i.e. the memory, we have shown link between the thermal memories of the quantum system by Generalized Fourier equation.

In a nutshell we implied relationship between Consciousness and Physics by showing a link between brain waves and quantum mechanics (i.e. link between thermal memory of a system and memory of human brain).

Indeed we humans are not yet at the level of understanding the consciousness, but as we keep contemplating it, we discover its very counterintuitive properties, and now we are a little closer towards the greatest leap i.e. consciousness is not just a subject of neuroscience, rather it is the new doctrine emerging from the shadows of Bio-physics.

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