Sound from von Kármán's vortex street

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Abstract

It is shown that the Strouhal friction tones are generated by von Kármán's vortex street. The non-relativistic and relativistic Strouhal number is derived from von Kármán's vortex street, where the relativistic formula follows from the addition law for velocities. The Strouhal friction tones are generated also by the motion of cosmic rays in relic photon sea, by motion of bolids in atmosphere, by the Saturn rings motion in the relic black-body sea, or by motion of proton bunches in LHC in CERN.

It is well known from the experimental artillery physics and ballistics, that the moving ballistic projectile generates not only the Mach cone but also the sound. Similarly, the moving projectile of a gun, moving bolid in atmosphere, moving tactic and ballistic misails generate sound. The physical origin of this sound is not caused by the vibration of the surface of the projectile, or by the vibration of the Mach cone, or by the micro-structure of the Mach cone, but it is caused by the periodic generation of vortexes in the vicinity of the surface of the projectile during the air flowing around it. Such sound is generated also by the air flow around the cylinders, or strings. The system of strings generating the sound is named Aeolean's harp (Aeolus being God of winds in the Greek mythology) and the tones generated in a such a way are so called the Strouhal friction tones. If the diameter of the string, or cylinder immersed in the flow is D and the velocity of the flow is v then the frequency f of the sound is given by the Strouhal formula:

$$f = \kappa(Re)\frac{v}{D}, \tag{1}$$

where κ is the Strouhal number named after Vincent Strouhal, a Czech physicist who experimented in 1878 with wires experiencing vortex shedding and singing in the wind (Strouhal, 1878; White, 1999). The symbol Re is the Reynolds number given by the formula $Re = vD/\nu$, where ν is the kinematic viscosity. The Strouhal number was late generalized to involve obertons, or

$$f = \kappa(Re)\frac{v}{D}n, \tag{2}$$

where n is the integer number of the oberton.

The von Kármán vortex street is named after the engineer and fluid dynamicist Theodore von Kármán (1963; 1994). It is produced for instance by wind interacting with the suspended telephone or power lines, or by a car antenna at certain speeds of a car.

The potential flow of the ideal liquid can be described by the complex function $w(z) = \varphi(x, y) + \psi(x, y)$ with $z = x + iy$ (Kočin et al., 1963).

The vortex potential of the one vortex of the fluid motion is

$$w(z) = \frac{\Gamma}{2\pi i}\ln\frac{(z - z_k)}{l}, \tag{3}$$

where Γ is so called circulation of liquid and l is the arbitrary constant with the dimensionality of length.

If we suppose many vortexes with centers at points $z_0, \pm z_1, \pm z_2, \pm z_3, ...$ with coordinates $x_k = lk, k = 0, \pm 1, \pm 2, \pm 3, ...$ and $y_k = H/2$, where H is the arbitrary parameter, then it may be easy to see that the complex potential of such system of vortexes is (Kočin et al. 1963):

$$w(z) = \frac{\Gamma}{2\pi i}\left\{\ln\frac{(z - z_0)\pi}{l} + \sum_{k=1}^{\infty}\left[\ln\frac{(z - z_k)}{-lk} + \ln\frac{(z - z_{-k})}{lk}\right]\right\} + const, \tag{4}$$

where we have multiplied $z - z_0$ by π/l and $z - z_k$ by $1/(-kl)$, which leads to the change of additional constant in the complex potential and not to the change of physics following from the complex potential.

After some mathematical manipulations, we get the last formula in the following form:

$$w = \frac{\Gamma}{2\pi i}\ln\sin\frac{\pi}{l}(z - z_0). \tag{5}$$

The corresponding complex velocity is as follows:

$$v_x - iv_y = \frac{\Gamma}{2li}\cot\frac{\pi}{l}(z - z_0). \tag{6}$$

In case of two vortexes with circulation Γ_1, Γ_2, we get the complex velocities in the form:

$$v_x - iv_y = \frac{\Gamma_1}{2li}\cot\frac{\pi}{i}(z - z_1) + \frac{\Gamma_2}{2li}\cot\frac{\pi}{l}(z - z_2) = -\frac{dw}{dz}. \tag{7}$$

It possible to see that for the interaction vortexes in line one and two, it is (Kočin et al., 1963):

$$v_{1x} - iv_{1y} = \frac{\Gamma_2}{2li}\cot\frac{\pi}{l}(z_1 - z_2) \tag{8}$$

and

$$v_{2x} - iv_{2y} = -\frac{\Gamma_1}{2li}\cot\frac{\pi}{l}(z_1 - z_2). \tag{9}$$

We have in case of the line stability of vortexes, equal complex velocities:

$$v_{1x} - iv_{1y} = v_{2x} - iv_{2y}, \tag{10}$$

or, the following evident relation

$$\Gamma_1 = -\Gamma_2. \tag{11}$$

In case that y-distances of vortexes are constant, then $v_{1y} = v_{2y} = 0$, and

$$z_1 - z_2 = b + Hi, \tag{12}$$

where b, H are some constants.

Now, let us use the formula:

$$\cot \frac{\pi}{l}(b + Hi) = \frac{\sin \frac{2\pi b}{l}}{\cosh \frac{2\pi H}{l} - \cos \frac{2\pi b}{l}} - i \frac{\sinh \frac{2\pi H}{l}}{\cosh \frac{2\pi H}{l} - \cos \frac{2\pi b}{l}}, \tag{13}$$

Then, it follows from the last equation that

$$v_{1,2x} = \frac{\Gamma}{2l} \frac{\sinh \frac{2\pi H}{l}}{\cosh \frac{2\pi H}{l} - \cos \frac{2\pi b}{l}}, \tag{14a}$$

$$v_{1,2y} = -\frac{\Gamma}{2l} \frac{\sin \frac{2\pi b}{l}}{\cosh \frac{2\pi H}{l} - \cos \frac{2\pi b}{l}}. \tag{14b}$$

We have from $v_{1y} = v_{2y} = 0$, that

$$\sin \frac{2\pi b}{l} = 0, \tag{15}$$

or, $b = 0$, $b = l/2$.

The situation with $b = 0$ is called the symmetrical configuration which is non-stable (Kočin et al., 1963) and the situation with $b = l/2$ which is the chess stable configuration. We have two velocities:

$$v_{1x} = \frac{\Gamma}{2l} \coth\left(\frac{\pi H}{l}\right); \quad (b = 0), \tag{16}$$

$$v_{2x} = \frac{\Gamma}{2l} \tanh\left(\frac{\pi H}{l}\right); \quad (b = l/2). \tag{17}$$

The period forming by the vortex street, where the relative velocities is $v - u$, is (Blokhintsev, 1981):

$$T = \frac{l}{v - u} \tag{18}$$

and the frequency f is

$$f = \frac{v - u}{l} = \left(1 - \frac{u}{v}\right) \frac{D}{l} . \frac{v}{D} \tag{19}$$

It means in the last formula that the non-relativistic Strouhal number κ is

$$\kappa = \left(1 - \frac{u}{v}\right)\frac{D}{l}. \tag{20}$$

The rigorous derivation of the relativistic Strouhal number follows from the relativistic hydrodynamics (Landau et al. 1987), together with the derivation of the relativistic von Kármán's vortex theory. However, we here suppose that the relativistic Strouhal nummber follows immediately from the non-relativistic formula by the operation of the relativistic generalization.

The Strouhal formula contains quantity D with the dimensionality of length, and velocities v and u. According to special theory of relativity, length is not contracted when the cylinder or string is placed perpendicular to the direction of motion, and it means that it is not contracted if it is placed perpendicular to the air flow in the considered experiment. On the other hand, the special relativity addition theorem is necessary to apply for velocities v and u. In other words the relativistic formula is as follows (with $v \oplus u$ being the relativistic addition) :

$$v \oplus u = \frac{v+u}{1+\frac{uv}{c^2}}. \tag{21}$$

Using the formula (25) for non-relativistic frequency generated by the vortexes, we get after some algebraic operations, the relativistic Strouhal number in the form:

$$\kappa = \frac{\left(1 - \frac{u}{v}\right)\frac{D}{l}}{\left(1 - \frac{uv}{c^2}\right)}. \tag{22}$$

Let us remark, that if we consider the Strouhal effect in the inertial system moving with velocity V with regard to the laboratory system, then it is necessary still transform the last formula according the relativistic Doppler formula.

We have considered the aerodynamic and hydrodynamical situations where the Strouhal friction tones are generated. The non-relativistic and relativistic Strouhal number was derived from so called von Kármán's vortex street. The relativistic derivation of this formula followed from the relativistic addition formula for velocities.

The physical phenomenon called the Strouhal friction tones can be extended to the cosmic rays moving in the relic photon sea, motion of bolids in atmosphere and the ionospheric generation of sound escorting the aurora borealis/australis. In case of cosmic rays we consider the moving bunch of cosmical particles with its effective diameter D and not the individual particles. The detection of the generated sound is possible by special microphones. Motion of bodies in the solar wind produces also the von Kármán vortex street leading to the Strouhal friction tones. During the motion of such bunch, the von Kármán photon vortex street is generated to form the Strouhal friction tones.

The moving bunch of protons with effective diameter D in the accelerated tube in LHC of CERN generates also the Strouhal friction tones, because the tube is the black-body with the photon sea (Pardy, 2013a; ibid., 2013b), which enables the formation of the von Kármán photon vortex street.

The Strouhal friction tones are also generated by submarine during the formation of the von Kármán phonon vortex street.

The cylinder of the diameter D immersed in the flame perpendicularly to the flame flow generates also the Strouhal friction tones.

The Saturn rings $R_i, i = 1, 2, ...$ are composed from massive objects with diameters $D_{i1}, D_{i2}, D_{i3}, ...$, moving in the relic photon see (Pardy, 2013a; ibid., 2013b) and producing the Strouhal friction tones. In such a way, the Saturn rings form the Saturn Aeolian harp in our planetary system. It is not excluded that the Strouhal friction tones of the Saturn rings can be detected by the special microphones of Bell laboratories.

The von Kármán vortexes are generated also in the Earth atmosphere , or in the Jupiter atmosphere, or, in other planetary atmospheres. It is not excluded that in cosmological space, the von Kármán vortexes are generated during formation of galaxies , however, if and only if the hydrodynamical limit of cosmological matter is possible.

Let us remark in conclusion that the von Karman vortexes and friction tones are also generated by the motion of bodies in liquid helium, which can be easily verified by the experiments in the low temperature laboratories.

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