Principle of Least Angular Action

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Abstract

This paper presents the principle of least angular action.

Principle of Least Angular Action

If we consider a single particle A of mass m_a then the principle of least angular action, is given by:

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2}\, m_a (\mathbf{r} \times \mathbf{v}_a)^2 \, dt + \int_{t_1}^{t_2} (\mathbf{r} \times \mathbf{F}_a) \cdot \delta (\mathbf{r} \times \mathbf{r}_a) \, dt = 0$$

where $\mathbf{r}$ is a position vector which is constant in magnitude and direction, $\mathbf{v}_a$ is the velocity of particle A, $\mathbf{F}_a$ is the net force acting on particle A, and $\mathbf{r}_a$ is the position of particle A.

If $-\delta V_a = (\mathbf{r} \times \mathbf{F}_a) \cdot \delta (\mathbf{r} \times \mathbf{r}_a)$ and since $T_a = \tfrac{1}{2}\, m_a (\mathbf{r} \times \mathbf{v}_a)^2$, then:

$$\delta \int_{t_1}^{t_2} (T_a - V_a) \, dt = 0$$

And since $L_a = T_a - V_a$, then we obtain:

$$\delta \int_{t_1}^{t_2} L_a \, dt = 0$$