Absolute parallelism, modified gravity, and suppression of gravitational *short* waves

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There is a unique variant of Absolute Parallelism, which is very simple as it has no free parameters: nothing (nor D=5) can be changed if to keep the theory safe from emerging singularities of solutions. On the contrary, eternal solutions of this theory, due to the linear instability of the trivial solution, should be of great complexity which can in some scenarios (with a set of slowly varying parameters of solutions) provide a few phenomenological models including a modified (better to say, new or another) gravity and an expanding-shell cosmology (the longitudinal polarization gives the anti-Milne model). The former looks (mostly) like $R^{\mu\nu}G_{\mu\nu}$-gravity on a brane of a huge scale L along the extra-dimension.

The correction to Newton's law of gravity, which depends in this theory on two parameters (bi-Laplace equation) and behaves as $1/r$ on large scales, $r > L$ (kpc $> L >$ pc), can start from zero (the Rindler term vanishes) if a constraint is imposed on these parameters. On further consideration, one can conclude that generation of gravitational 'short' waves, $\lambda < L$, is inhibited in this new gravity.

1. Introduction

The basement of the modern physics is composed of two main theories: the general relativity theory (GR; the leading and supposedly successful, at the expense of inventing those dark entities, gravitation theory) and the standard model of elementary particle physics (a variant of quantum field theory which has a great number of unexplained features and fitting parameters). These two are based on very different principles and even symmetries, but they have something in common: both are affected by the problem of singularities or divergencies (despite different shifts and tricks, like supersymmetry, strings, and so on) of solutions. It is generally agreed that a more fundamental theory should unite these two branches, two domains of natural phenomena; and it is deadly evident that a reasonable theory should be free from singularities of solutions.[1]

There are too many interpretations of quantum mechanics;[2] two of them – Copenhagen and many-worlds – are of pretty similar 'rating' (this means that none is convincing enough).

Einstein was not satisfied with GR (and quantum mechanics as well) and he had proposed absolute parallelism (AP) which unites symmetries of both general and special relativity theories. Einstein and Mayer had obtained a vast list of compatible second order equations of ($4D$) AP [1], most of them are non-Lagrangian; however this list is a bit incomplete.

Theoretical physicists form in fact a quite specific subset of experimentalists: they are doing experiments on their own brains. The mainstream theorists participate in highly collective 'experiments'. However, as one should note, string theory, as well as M-theory, still does not deserve the definite article, *the*. (That is, nobody can answer, what is string theory?)

The result of my own 'experiment' is a single-field theory, really simple (according to Kolmogorov's theory of algorithm complexity) and beautiful (that is, of very large symmetry) – the unique (no free parameters) $5D$ variant of Absolute Parallelism, which is free from emerging singularities in solutions of general position [2]. (The "Little Prince's Principle" states: true beauty should be single, should be unique. A theory with free parameters, or even a single parameter, is actually a huge set of slightly different theories, only one of which is supposedly true, but that true value(s) will never be measured "exactly", at least because of funding limitations; and there are no reasons to see "less beauty" in all other, wrong variants of such a theory. Phenomenological models, with a set of 'free parameters', of course, are of some usefulness, but

[1] Some gauge dials carry the infinity sign ∞, but it's just an exaggeration – no one can measure infinity.

[2] The most weird one imputes free will to elementary particles (= different tiny spaceship models); see arXiv:physics/0004047 by R. Nakhmanson; sure, the degree of arbitrariness and unknownness in this model is some greater than in string theory, or in the strand model (motionmountain.net/research.html).

what we really need is a really fundamental theory which would provide us with all necessary and well-behaved phenomenologies.)

Interestingly, in the absence of singularities, AP obtains topological features a tad similar to that of nonlinear sigma-models. In order to give a clear presentation and full picture of the theory, many items should be sketched: linear instability of the trivial solution and expanding O_4-symmetrical solutions; tensor $T_{\mu\nu}$ (positive energy, but only three polarizations of 15 carry D-momentum and angular momentum; how to quantize ?) and post-Newtonian effects; topological classification of symmetrical $5D$ field configurations (alighting on evident parallels with the particle combinatorics and chiral patterns of the Standard Model) and a 'quantum phenomenology on an expanding classical background'; a 'phenomenological' $R_{\mu\nu}G^{\mu\nu}$-gravity on a very thick brane and a change in the Newton's Law: $\frac{1}{r^2}$ goes to $\frac{1}{r}$ with distance. (This is different from the MOND paradigm [3]. MOND is not free of a strangeness: given two bodies of very different mass, one can choose the distance between them such that the heavier body is in the MOND regime, while the other still in the Newtonian regime – as a result, the third Newton's law, if not the second which is testable [4], should be violated. Moreover, MOND means violation of linearity exactly for the case of small accelerations.)

The linear instability of the trivial solution, a very interesting feature, makes inevitable the existence of strong non-linear effects; interestingly, the unstable waves (growing polarizations) do not contribute to the energy-momentum. On the other hand, a spherically-symmetrical single wave running along the radius (the longitudinal polarization, by itself, is stable) can serve as a region of enhanced instability, a kind of wave-guide with a specific (ultrarelativistic) reduction of the extra-dimension. A Lagrangian phenomenology of topological quanta (a kind of topological Brownian particles which carry topological charges and/or quasi-charges [2, 6]) emerges, which can look like a Quantum Field Theory (although the underlying theory is absolutely classical !).

The theory seems able to easily explain the meaning of many features of the Standard Model – the flavors and colours, quark confinement, ubiquity of the least action principle (and the very superposition principle as well). Moreover, this theory gives a number of testable predictions:

- spin zero elementary particles do not exist;
- neutrinos are true neutral (a kind of Majorana);
- there is no room for SUSY, Dark Matter, and Dark Energy;
- additional pseudovector bosons (responsible for dynamical mass generation) can exist;
- the gravitational part of the Lagrangian is $R^{\mu\nu}G_{\mu\nu}$, and, due to the large extra-dimension, it gives switching from Newton's law, $1/r^2$ (at small distances), to the more slowly decreasing force, $1/r$, at larger distances;
- the Hubble plot should be described by the anti-Milne model [in FRW-framework it means $a = a_0(1 + H_0 t), k = +1$], without any fine tuning and free parameters (excepting the Hubble constant, of course).

Frankly, at the present time, this theory has seemingly some more reasons, than any other existing one, for a belief that it is *on the right track*. Here I am going to add one more qualitative prediction relating to the subject of gravitational waves:

- generation of gravitational 'short' waves ($\lambda<L$) is suppressed.

But first let's dwell a bit on polarization degrees of freedom in GR and modified gravities.

2. AP vs GR and Riemann-squared gravities; polarization degrees

The case of vacuum GR is a simple example of a single-field theory – the components of the metric field transform as a single irreducible representation; also it is a special, degenerate case of AP. Symmetries of the vacuum equation of GR, besides the local group of coordinate diffeomorphisms, Diff(D), include the global symmetry – global conform transformations.

If $g(x)$ is a solution, than $\kappa^2 g(x)$ (where κ=const) is also a solution. This simple symmetry leads to (or explains) the notion of [length] dimension which is really a global feature.[3] Accompanying this, $g_{\mu\nu} \to \kappa^2 g_{\mu\nu}$, with the scale change of coordinates, $x^\mu \to \kappa x^\mu$, one easily obtains

$$g_{\mu\nu} \to \kappa^0 g_{\mu\nu}; \; g^{\mu\nu} \to \kappa^0 g^{\mu\nu}; \; g_{\mu\nu,\lambda} \to \kappa^{-1} g_{\mu\nu,\lambda}; \;\; \text{et cet.} \tag{1}$$

The power of factor κ corresponds to the power of *length* indicating the dimension of one or another value. The moral is that global symmetries could also be very important (and intention to introduce the local conform symmetry is not so reasonable). In the presence of the cosmological constant, the GR equation is no longer homogeneous, and the global conform symmetry disappears.

There is, however, another noteworthy global feature in nature — the signature of spacetime. In special relativity, it follows from the global symmetry, Lorentz group, but it has no such a 'symmetry substantiation' in the case of general relativity. The absolute parallelism theory (AP) improves the situation. The frame field of AP, $h^a_{\ \mu}$, is just a square matrix with indexes of different nature; it admits both local coordinate transformations (act on Greek indices) and global transformations of 'extended' Lorentz group (the point symmetry group of inertial coordinates; act on Latin indices; AP is sure a single field theory):

$$h^{*a}{}_\mu(y) = \kappa s^a{}_b h^b{}_\nu(x) \partial x^\nu / \partial y^\mu; \quad \kappa > 0, \quad s^a{}_b \in O(1, D-1), \quad s^a{}_b, \kappa = \text{const}. \tag{2}$$

The metric here is just the next quadratic form on the basis of the Minkowski metric:

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu, \quad \eta_{ab}, \eta^{ab} = \text{diag}(-1, 1, \ldots, 1)\,.$$

Field equations of such a great symmetry can be composed using a covariant notation, that is, the usual covariant differentiation with symmetric Levi-Civita connection, ';', and the fundamental tensor of this theory (which is a way simpler than the curvature tensor; coma ',' denotes the usual partial derivative, while square brackets denote anti-symmetrization of indexes):

$$\Lambda_{a\mu\nu} = h_{a\mu,\nu} - h_{a\nu,\mu} = 2h_{a[\mu;\nu]} \;\; (g_{\mu\nu;\lambda} \equiv 0); \; \Lambda_{a[\mu\nu;\lambda]} \equiv 0\,. \tag{3}$$

This tensor has three irreducible parts, including vector and skew-symmetric tensor of rank three (sometimes, in a covariant context, we omit in contractions matrices $\eta^{ab}, \eta_{ab}, g_{\mu\nu}, g^{\mu\nu}$ — there they can be restored unambiguously):

$$\Phi_a = \eta_{bc} \Lambda_{b\mu\nu} h_c{}^\mu h_a{}^\nu = \Lambda_{bba}, \quad S_{abc} = 3\Lambda_{[abc]} = \Lambda_{abc} + \Lambda_{bca} + \Lambda_{cab}\,. \tag{4}$$

The simplest derivative covariant in GR is the Riemann tensor which has three irreducible parts: Ricci scalar R, Ricci tensor $R_{\mu\nu}$, and Weyl tensor; only the last is responsible for gravitational waves (polarizations) as the others are 'fixed' by the field equations of GR.

The situation is different in Riemann-squared gravities with a Lagrangian composed of the three invariants quadratic in the Riemannian curvature; however, due to different reasons, these modified gravities are all unappropriate [as well as $f(R)$-gravities, $f(R) \neq R$].

For example, $(a + bR + R^2/2)$-gravity leads to an incompatible system of equations (the trace part, $\mathbf{E}_\mu{}^\mu = 0$, can be used that to remove the term with $R_{;\lambda;\lambda}$ — excepting the case $D = 1$):

$$\mathbf{E}_{\mu\nu} = R_{;\mu;\nu} - R_{\mu\nu}(b + R) + g_{\mu\nu}(a/2 + bR/2 + R^2/4 - R_{;\lambda;\lambda}) = 0, \tag{5}$$

[3] Length is so in Africa as well (i.e., dlina – ona i v Afrike dlina).

$$\mathbf{E}^{\star}_{\mu\nu} = \mathbf{E}_{\mu\nu} + \varkappa\, g_{\mu\nu}\mathbf{E}_{\lambda}{}^{\lambda} = R_{;\mu;\nu} - R_{\mu\nu}(b+R) + g_{\mu\nu} f^{\star}(R) = 0.$$

The next combination of prolonged equations, $\mathbf{E}^{\star}_{\mu\nu;\lambda} - \mathbf{E}^{\star}_{\mu\lambda;\nu} = 0$, after cancellation of the principal derivatives (5-th order), gives new 3-d order equations which are irregular in the second jets: the term $R_{;\varepsilon}R^{\varepsilon}{}_{\mu\nu\lambda}$ can not be cancelled by the other terms which contain only the Ricci tensor and scalar. The rank of these subsystem depends on the second derivatives, $g_{\mu\nu,\lambda\rho}$ (see [5] for the definition of PDE's regularity).

The most interesting case, $R_{\mu\nu}G^{\mu\nu}$-gravity (the Ricci tensor is contracted with the Einstein tensor), gives the following compatible system:

$$-\,\mathbf{D}_{\mu\nu} = G_{\mu\nu;\lambda}{}^{;\lambda} + G^{\epsilon\tau}(2R_{\epsilon\mu\tau\nu} - \tfrac{1}{2}g_{\mu\nu}R_{\epsilon\tau}) = 0;\;\; \mathbf{D}_{\mu\nu;\lambda}g^{\nu\lambda} \equiv 0\,. \tag{6}$$

In linear approximation, there are simple evolution equations for the Ricci tensor and scalar: $\Box R = 0$, $\Box R_{\mu\nu} = 0$. Using the Bianchi identity, $R_{\mu\nu[\lambda\epsilon;\tau]} \equiv 0$, its prolongation and contractions,

$$R_{\mu\nu[\lambda\epsilon;\tau];\rho}g^{\tau\rho} \equiv 0,\;\; R_{\mu\nu[\lambda\epsilon;\tau]}g^{\mu\tau} \equiv 0,$$

we write the evolution equation of Riemann tensor (we'll need just the linear approximation):

$$R_{\mu\epsilon\nu\tau;\rho;\rho} = R_{\mu\nu;(\epsilon;\tau)} - R_{\mu\tau;(\nu;\epsilon)} + R_{\epsilon\tau;(\mu;\nu)} - R_{\epsilon\nu;(\mu;\tau)} + (\text{Riem.}^2). \tag{7}$$

This equation is more complex: it has a linear source term (in its RHS) composed from the Ricci tensor. As a result, in the general case when the Ricci-polarizations do not vanish, the polarizations relating to the Weyl tensor [and responsible for gravitational waves and tidal forces; their number is usual $D(D-3)/2$ plus one extra ('spin zero' or 'trace') polarization] should grow linearly with time,

$$a(t) = (c_0 + c_1 t)e^{-i\omega t},$$

while the linear approximation is valid. The total number of polarizations is $(D-1)^2$ – forth order equations have much more voluminous a Cauchy problem. There are no imaginary frequencies, no exponentially growing 'eigen vectors' (exponential growing would contradict the correctness of the Cauchy problem), or polarizations – all eigen values are just $w^2 = k^2$, but some of them are doubly degenerate, and some amplitudes should linearly grow.

This means that the regime of weak gravity is linearly unstable, as well as the trivial solution itself (*i.e.*, in this theory, *nothing* is not so *real*). Hence, this theory is physically irrelevant, because we are still living in conditions of very weak gravity. [Note that in General Relativity, when the Ricci tensor is expressed through the energy-momentum tensor (which does not expand into plane waves – with the dispersion law of light in a vacuum), equation (7) describes the process of gravitational wave generation.]

This linear instability does not contradict the correctness of Cauchy problem; the compatibility theory (Kovalevskaya's theorem and its generalization; see Pommaret's book [5]) gives easy answers about the Cauchy problem, number of polarizations, and so on (especially easy for analytical PDE systems).

The third possible term (see, e.g., [2]), can be written [using 5-minor of the metric, minor of corank five; the corresponding Lagrangian, Gauss-Bonnet or Lovelock term, can be written using 4-minor $[\mu\nu,\alpha\beta,\gamma\delta,\varepsilon\tau] \equiv \partial^4(-g)/(\partial g_{\mu\nu}\partial g_{\alpha\beta}\partial g_{\gamma\delta}\partial g_{\varepsilon\tau})$] as

$$\mathbf{D}^{\mu\nu}_{(3)} = (-g)^{-1}[\mu\nu,\alpha\beta,\gamma\delta,\varepsilon\tau,\rho\phi]R_{\alpha\gamma\varepsilon\rho}R_{\beta\delta\tau\phi}\,;\;\; \mathbf{D}^{\mu\nu}_{(3)\,;\nu} \equiv 0\,.$$

Covariant differentiation of the minor divided by $(-g)$ is identically zero (only metric $g^{\mu\nu}$ is there), while differentiation of either Riemann tensor leads to application of Bianchi identity (due to contraction with a highly skewsymmetric – separately, in the 'row indexes', and in the 'column indexes' – tensor). This tensor identically vanishes if $D \le 4$ (some people called this a 'tricky identity', if I am not mistaken): $\mathbf{D}^{\mu\nu}_{(3)} \equiv 0$, because 5-minor is identically zero in low dimensions (you should cross out five rows and five columns, but that is impossible when the metric is just a 4x4 matrix).

In higher dimensions, if this tensor is the main term, the equations are irregular in second jets (and hence unappropriate). If this term is an addition (to a 4-th order Ricci-squared gravity or $R_{\mu\nu}G^{\mu\nu}$-gravity), it does not change the conclusion about the linear instability of the Weyl polarizations (or do not cure the irregularity of the Ricci-scalar-squared gravity).

As regards AP, the simplest compatible second order system (non-Lagrangian, as it does not contain a term like $h_{a\mu}L$)

$$\mathbf{E}^*_{a\mu} = \Lambda_{a\mu\nu;\nu} = 0 \text{ [i.e. } (h\Lambda_a{}^{\mu\nu})_{,\nu} = 0,\ h = \det h^a{}_\mu = \sqrt{-g}\,;\ \ \mathbf{E}^*_{a\mu;\mu} \equiv 0\,] \tag{8}$$

looks, after linearization, like a D-fold Maxwell's equation, see eq. (3), where infinitesimal diffeomorphisms serve as a set of gauge transformations; so, the number of polarisation degrees of freedom in this case (as well as for other AP equations with similar identities) is $D(D-2)$.

3. Co- and contra-singularities in AP, and the unique field equation

AP is more appropriate as a modified gravity, or just a good theory with topological charges and quasi-charges (their phenomenology, at some conditions and to the certain extent, can look like a quantum field theory) [2, 6].

There is one unique equation of AP (non-Lagrangian, with a unique D) which solutions are free of arising singularities. The formal integrability (compatibility) test [5] can be extended to the cases of degeneration of either co-frame matrix, $h^a{}_\mu$ (co-singularities), or contra-variant frame (or frame density of some weight), serving as a local and covariant test for singularities. This test singles out the next, unique equation (and $D=5$ [2]; see eq. (3); $h = \det h^a{}_\mu = \sqrt{-g}$):

$$\mathbf{E}_{a\mu} = L_{a\mu\nu;\nu} - \tfrac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_\nu) = 0\,; \tag{9}$$

here $$L_{a\mu\nu} = L_{a[\mu\nu]} = \Lambda_{a\mu\nu} - S_{a\mu\nu} - \tfrac{2}{3}h_{a[\mu}\Phi_{\nu]},\ f_{\mu\nu} = 2\Phi_{[\mu;\nu]} = \Phi_{\mu,\nu} - \Phi_{\nu,\mu}.$$

One should retain the identities (for further details see [2, 6]):

$$\Lambda_{a[\mu\nu;\lambda]} \equiv 0\,,\quad h_{a\lambda}\Lambda_{abc;\lambda} \equiv f_{cb}\,(= f_{\mu\nu}h^\mu_c h^\nu_b),\ \ f_{[\mu\nu;\lambda]} \equiv 0. \tag{10}$$

Equation $\mathbf{E}_{a\mu;\mu} = 0$ gives a *Maxwell-like* equation: $(f_{a\mu} + L_{a\mu\nu}\Phi_\nu)_{;\mu} = 0$,

$$\text{or}\ \ f_{\mu\nu;\nu} = (S_{\mu\nu\lambda}\Phi_\lambda)_{;\nu}\ \ [= -\tfrac{1}{2}S_{\mu\nu\lambda}f_{\nu\lambda},\ \text{see eq-n (12) below}]\,. \tag{11}$$

In reality, eq-n (11) follows from the symmetric part only, because the skewsymmetric one gives an identity; note also that the trace part becomes irregular if $D=4$ (forbidden D; the principal derivatives vanish):

$$2\mathbf{E}_{[\nu\mu]} = S_{\mu\nu\lambda;\lambda} = 0,\ \mathbf{E}_{[\nu\mu];\nu} \equiv 0; \tag{12}$$

$$\mathbf{E}_{\mu\mu} = \mathbf{E}_{a\mu}h^\mu_b\eta^{ab} = \tfrac{4-D}{3}\Phi_{\mu;\mu} - \tfrac{1}{2}\Lambda^2_{abc} + \tfrac{1}{3}S^2_{abc} + \tfrac{D-1}{9}\Phi^2_a = 0. \tag{13}$$

System (9) remains compatible under adding $f_{\mu\nu} = 0$, see (11); this is not the case for the other covariants, S, Φ, or the Riemann curvature; the last relates to tensor Λ as usually:

$$R_{a\mu\nu\lambda} = 2h_{a\mu;[\nu;\lambda]};\ h_{a\mu}h_{a\nu;\lambda} = \tfrac{1}{2}S_{\mu\nu\lambda} - \Lambda_{\lambda\mu\nu}.$$

GR is a special case of AP. Using 3-minors (corank-3), $[\mu\nu, \varepsilon\tau, \alpha\beta] \equiv \partial^3(-g)/(\partial g_{\mu\nu}\partial g_{\varepsilon\tau}\partial g_{\alpha\beta})$, and their skew-symmetry features, one can write the vacuum GR equation as follows:

$$2(-g)G^{\mu\nu} = [\mu\nu, \varepsilon\tau]_{,\varepsilon\tau} + (g'^2) = [\mu\nu, \varepsilon\tau, \alpha\beta](g_{\alpha\beta,\varepsilon\tau} + g^{\rho\phi}\Gamma_{\rho,\varepsilon\tau}\Gamma_{\phi,\alpha\beta}) = \tfrac{1}{2}[\mu\nu, \varepsilon\tau, \alpha\beta]R_{\alpha\varepsilon\tau\beta} = 0. \tag{14}$$

Similarly, all (but one) AP equations can be reshaped in such a way that 2-minors of co-frame,

$$\binom{\mu\ \nu}{a\ b} = \frac{\partial^2 h}{\partial h^a{}_\mu \partial h^b{}_\nu} = 2!\, h h_{[a}{}^\mu h_{b]}^\nu, \text{ i.e.,} \quad [\mu_1\nu_1, \ldots, \mu_k\nu_k] = \frac{1}{k!}\binom{\mu_1 \cdots \mu_k}{a_1 \cdots a_k}\binom{\nu_1 \cdots \nu_k}{a_1 \cdots a_k},$$

completely define the coefficients at the principal derivatives.

For example, the simple equation (8) gives [2]

$$h^2 \mathbf{E}^{*\mu}_a = -g g^{\alpha\mu} g^{\beta\nu}(h_{a\alpha,\beta\nu} - h_{a\beta,\alpha\nu}) + \cdots = h_{a\alpha,\beta\nu}[\alpha\mu, \beta\nu] + (h'^2)\ .$$

Like the determinant, k-minors ($k \le D$) are multi-linear expressions in elements of co-frame matrix, $h^a{}_\mu$, and some minors do not vanish when $\operatorname{rank} h^a{}_\mu = D-1$.

For any AP equation [including eq-ns (14) and (8)], with the *unique exception*, eq. (9), (where only the skew-symmetric part participates in the identity and can be written with 2- and 3-minors, while the symmetric part needs 1-minors which vanish too simultaneously when the co-frame matrix degenerates), the principal terms keep regularity (and the symbol G_2 remains involutive [2]) if $\operatorname{rank} h^a{}_\mu = D-1$. This observation is important and relevant to the problem of singularities; it means seemingly that the unique equation (9) does not suffer of co-singularities in solutions of general position.

The other case, contra-singularities [2], relates to degeneration of a contravariant frame density of some weight:

$$H_a{}^\mu = h^{1/D_*} h_a{}^\mu; H = \det H^a{}_\mu,\ h_a{}^\mu = H^{1/(D-D_*)} H_a{}^\mu\ . \tag{15}$$

Here D_* depends on the choice of equation: $D_* = 2$ for GR, $D_* = \infty$ for eq-n (8), and $D_* = 4$ for the unique equation (which can be written 3-linearly in $H_a{}^\mu$ and its derivatives [2]).

If integer, D_* is the forbidden spacetime dimension. For the unique equation, the nearest possible D, $D = 5$, is of special interest: in this case minor $H^{-1} H^a{}_\mu$ simply coincides with $h^a{}_\mu$; that is, a contra-singularity simultaneously implies a co-singularity (of high corank), but that is impossible! The possible interpretation of this observation is: for the unique equation, contra-singularities are impossible if $D = 5$ (perhaps due to some specifics of *Diff*-orbits on the $H_a{}^\mu$-space). This leaves no room for any changes in the theory (if nature abhors singularities).

4. Stress-energy tensor and new gravity with a 'weak Lagrangian'; *dwarf*, *normal*, and *giant* (unstable) polarization degrees in AP

One can rearrange $\mathbf{E}_{(\mu\nu)} = 0$ picking out (into LHS) the Einstein tensor, but the rest terms are not a proper stress-energy tensor: they contain linear terms $\Phi_{(\mu;\nu)}$ [no positive energy (!)]:

$$\mathbf{E}_{(\mu\nu)} + 2g_{\mu\nu}\mathbf{E}_{\lambda\lambda} = -G_{\mu\nu} - \tfrac{2}{3}\Phi_{(\mu;\nu)} + (\Lambda^2\text{-terms}) = 0. \tag{16}$$

However, the prolonged equation $\mathbf{E}_{(\mu\nu);\lambda;\lambda}$ can be written as $R_{\mu\nu}G^{\mu\nu}$-gravity (6):

$$G_{\mu\nu;\lambda;\lambda} + G_{\epsilon\tau}(2R_{\epsilon\mu\tau\nu} - \tfrac{1}{2}g_{\mu\nu}R_{\epsilon\tau}) = T_{\mu\nu}(\Lambda'^2, \cdots),\ T_{\mu\nu;\nu} = 0; \tag{17}$$

up to quadratic terms, $T_{\mu\nu} = \frac{2}{9}(\frac{1}{4}g_{\mu\nu}f^2 - f_{\mu\lambda}f_{\nu\lambda}) + B_{\mu\epsilon\nu\tau}(\Lambda^2)_{,\epsilon\tau}$ [2]; tensor B has symmetries of the Riemann tensor, so term B'' adds nothing to the D-momentum and angular momentum.

This equation (17) follows also from the least action principle. The 'weak Lagrangian' (the term of N.Kh. Ibragimov for the case when variation is zero due to both field equations and their prolongations) is quadratic in the field equations, *i.e.* is trivial[4] [one should use the trace eq-n (13), and the identity $R = -2\Phi_{\mu;\mu} + (\Lambda^2)$; $D = 5$]:

$$L = \mathbf{E}^2_{(\mu\nu)} - 7\mathbf{E}^2_{\lambda\lambda} \equiv R_{\mu\nu}G^{\mu\nu} + \tfrac{1}{9}f^2_{\mu\nu} + \tfrac{4}{9}[(3G_{\mu\nu} - \Phi_{\mu;\nu})\Phi_\mu + \Phi_{\lambda;\lambda}\Phi_\nu]_{;\nu} + (\Lambda'\Lambda^2,\ \Lambda^4). \tag{18}$$

[4] This triviality, however, is of another sort than the triviality of surface terms.

The main, quadratic terms, after exclusion of covariant divergences (surface terms), look like a modified gravity (higher terms can add to $T_{\mu\nu}$ only a trivial quadratic contribution, like B''.

This Lagrangian is trivial, as well as all its Noether currents; this also means that the contribution of gravitation to the 'total energy' is negative and the 'total energy' is strictly zero. (All this Lagrangian issue follows as a mere bonus, without any *ad hoc* activity !)

Note that only f-covariant (three transverse polarizations in $5D$) carries D-momentum and angular momentum (*ponderable* or *tangible* waves); other 12 polarizations are *imponderable*, or *intangible*. This is a very strange thing (it is scarcely possible in the Lagrangian tradition).

These f-waves feels only the metric and S-field, see (11), but S has effect only on polarization ('spin') of these waves: $S_{[\mu\nu\lambda]}$ does not enter the eikonal equation, and f-waves moves along the usual Riemannian geodesics.

However, f-component is not the usual (quantum) EM-field, it's just an important covariant responsible for energy-momentum (there is no gradient invariance for f [2]).

Another important feature is the linear instability of the trivial solution: some *intangible* polarizations grow linearly with time in the presence of *tangible* f-waves. Really, the linearized eq-n (9) and identity (10) yield (the following equations should be understood as linearized):

$$3\Lambda_{abd,d} = \Phi_{a,b} - 2\Phi_{b,a} \text{ (trace part: } \Phi_{a,a} = 0), \;\; \Lambda_{a[bc,d],d} \equiv 0, \;\Rightarrow\; \Lambda_{abc,dd} = -\tfrac{2}{3} f_{bc,a}\,. \tag{19}$$

The last D'Alembert equation has a *source* in its RHS. Some components of Λ (most symmetrical irreducible parts, as well as the Riemann curvature) do not grow because (linearized equations again)

$$S_{abc,dd} = 0,\;\; \Phi_{a,dd} = 0,\;\; f_{ab,dd} = 0,\;\; R_{abcd,ee} = 0.$$

However the least symmetrical Λ-components (triangle Young diagram), in fact only three polarizations of them which are to be called $\Lambda^{\bullet}$-waves (three growing but intangible polarizations), do go up with time if the ponderable waves (three f-polarizations) do not vanish. This should be the case for solutions of general position. These *giant* polarizations, $\Lambda^{\bullet}$-waves, should result in strong nonlinear effects, and it is of special interest if some space regions can witness more f-waves and hence more instability, more nonlinearities, in comparison with other regions.

The forth polarization of vector Φ_μ [the fifth one is eliminated by the trace equation, (13)] is the (only) longitudinal polarization; it relates to the gradient part: $\Phi_\mu = \Psi_{,\mu}$. One can formally write the evolution equation for the longitudinal polarization [see eq-n (13)], $\Box\Psi = \Lambda^{\bullet 2} + \cdots$; so, the giant polarizations squared do influence the longitudinal polarization.

[Interestingly, the linearized equations (9) loose its trace part if $D = 4$ (forbidden dimension; still one can add eq-n $\Phi_{\mu,\mu} = 0$ 'by hand') and in this case there is a new symmetry — with respect to infinitesimal conform transformations which serve as a kind of gradient transformations of vector Φ_μ, and, therefore, eliminate the longitudinal polarization, so to say.]

The skew-symmetric tensor S is responsible for three polarizations. One can introduce pseudo-tensor (remember $D = 5$)

$$\tilde{f}_{ab} = \tfrac{1}{6}\,\varepsilon_{abcde} S_{cde};$$

then, from eq-n (12) and the totally skew-symmetric part of identity (10), it follows (again a Maxwell-like system):

$$\tilde{f}_{[\mu\nu;\lambda]} = 0\,,\;\; \tilde{f}^{\mu\nu}{}_{;\nu} = \tfrac{1}{8}\,h^{-1}\,\varepsilon^{\mu\nu\lambda\varepsilon\tau}\Lambda_{a\nu\lambda}\Lambda_{a\varepsilon\tau}\,.$$

So, we have just three S-polarizations.

Three $\Lambda^{\bullet}$-polarizations correspond to Λ-tensor of a specific, gradient (or rotor-gradient) form: $\Lambda_{\varepsilon\mu\nu} = A_{[\mu,\nu];\varepsilon}$. At last, there remain five polarizations; this is just the number of ordinary gravitational (Weyl) polarizations (in $5D$); the evolution of these waves, see eq-ns (7) and (16), again has $\Lambda^{\bullet 2}$-terms in its RHS (as a source) — this time organized as a tensor relating to the square Young diagram (symmetry of the Weyl tensor).

The current for the f-waves is just Sf-term, see (11), therefore these waves are most weak, *dwarf*, that is, their amplitude, a_f, should be smaller than the amplitudes of all other polarizations, $a_{\Lambda^\bullet} \gg a_W, a_S, a_L \gg a_f$ $(3_{\Lambda^\bullet} + 5_W + 3_S + 1_L + 3_f = 15)$.

If some form of reduction to a $4D$ picture takes place, there could come forth eight 'preferable for $4D$' (or not so sensible to the extra dimension) polarizations: $2_{\Lambda^\bullet} + 2_W + 1_S + 1_L + 2_f = 8$.

5. Expanding O_4-wave and cosmology; topological (quasi-)charges

The great symmetry of AP equations gives scope for symmetrical solutions. In contrast to GR, eq-n (9) has non-stationary spherically symmetric solutions (as an example of longitudinal waves). An O_4-symmetric field can be generally written [2] as

$$h^a{}_\mu(t, x^i) = \begin{pmatrix} a & b\,n_i \\ c\,n_i & e\,n_i n_j + d\Delta_{ij} \end{pmatrix}; \quad n_i = \frac{x^i}{r}; \qquad (20)$$

here $i, j = (1, 2, 3, 4)$, $a, \ldots, e$ are functions of time, $t = x^0$, and radius r, $\Delta_{ij} = \delta_{ij} - n_i n_j$, $r^2 = x^i x^i$.

As functions of radius, b, c are odd while the others even; the boundary conditions are: $e = d$ at $r = 0$, and $h^a{}_\mu \to \delta^a_\mu$ as $r \to \infty$. Placing in (20) $b = 0, e = d$ (another interesting choice is $b = c = 0$) and making integrations, one arrives to the next system (it resembles Chaplygin gas dynamics; dot and prime denote time and radius derivatives, respectively.)

$$\dot{A} = AB' - BA' + \tfrac{3}{r}AB, \;\; \dot{B} = AA' - BB' - \tfrac{2}{r}B^2, \qquad (21)$$

where $A = a/e = e^{1/2}$, $B = -\,c/e$. This system has non-stationary solutions, and a single-wave solution (of *proper sign*) might serve as a suitable (stable) cosmological expanding background. The condition $f_{\mu\nu} = 0$ is a must for solutions with such a high symmetry (as well as $S_{\mu\nu\lambda} = 0$); so, these O_4-solutions carry no energy, weight nothing — some lack of *gravity*!

A more realistic cosmological model might look like a single O_4-wave (or a sequence of such waves) moving along the radius and being filled with a sort of chaos, or an ensemble of chaotic waves, both tangible (*dwarf*; $a_f \ll 1$) and intangible ($a_{\Lambda^\bullet} < 1$, but intense enough that to give non-linear fluctuations with $\delta h \sim 1$). Development and examination of stability of this model is an interesting problem. The metric inhomogeneity in such a cosmological O_4-wave can serve as a slowly varying *shallow dielectric waveguide* for that dwarf f-waves [2]. The ponderable waves should have wave-vectors almost tangent to the S^3-sphere of the wave-front that to be trapped inside this spherical shell; the *giant* waves can grow up, and partly escape from the waveguide, and their wave-vectors can be some less tangent to the S^3-sphere. The shell thickness can be small for an observer in the center of O_4-symmetry, but in co-moving coordinates it can be very large, but still much smaller than the current radius of the spherical shell, $L \ll R$.

This picture leads to the anti-Milne cosmological model, $a = a_0(1 + H_0\,t)$, $k = +1$, with the next simple equation of distance modulus (it's good for SNe Ia/GRB data;[5] $d_* = 10\,\mathrm{pc}$):

$$\mu(z) = \mu_0 + 5\log[(1+z)\,\ln(1+z)], \;\; \mu_0 = -5\log(H_0 d_*/c) \approx 43.3\,.$$

This model does well the *job of inflation.* Only very small part of the spherical shell corresponds to $z_{\rm CMB} \sim 10^3$ (decoupling of CMB): $\varphi \simeq \frac{1}{\Gamma}\ln(1+z)$ (γ-factor $\Gamma \gg 1$), $\varphi_{\rm CMB} \ll 1$. Moreover, very separated, even opposite points of the shell (at such z) are not causally independent – they have the common past along the extra dimension.

The symmetry of this cosmological background is very high, enabling an interesting set of topological quasi-charges [localised field configurations of some (sub)symmetry, carrying a discrete feature — a topological quasi-charge], and some phenomenology of *topological quanta* on expanding, chaotic background should emerge. Because of time/volume limitations I will not settle in detail this subject (just see [2], [6]a). Still we should correlate somehow the 'true' (or

[5] See some diagrams in zhogin.narod.ru/pirt11.pps, or [6]b.

naturally geometrical) tensor $T_{\mu\nu}$ of eq. (17) (i.e. its *quantum* part which arises while topological quanta scatter and disturb the chaotic ensemble of perceptible f-waves) with the 'phenomenological energy-momentum' of GR. In units $\hbar = 1, c = 1$, the 'phenomenological momentum' of a particle is just its wave-vector, but the 'true' momentum, sure being proportional to the wave-vector of quantum's psi-function,[6] should include the small factor a_f^2, which defines the overall scale of the perceivable, true-energy carrying waves. So, a rude estimation is possible:

$$T_{\mu\nu} \approx a_f^2 T_{\mu\nu}^{(\text{phen.})}, \quad \text{and (see the next section)} \quad \lambda_{\text{Planck}}^2 \approx a_f^2 L^2 \,. \tag{22}$$

It seems that, in this theory, the Planck length is not of a fundamental sense (and the spectrum of chaotic waves should not continue to such a small wavelength).

6. Newton's gravity changes; suppression of short gravitational waves

A massive body in this theory (assuming that it is right) should be of great length along the extra dimension, and we would like to estimate the behavior of gravitational potential (the case of weak static field), and possible deviation from Newton's law of gravity.

Let us start with a point mass; the 'new gravity', eq. (17), gives a 4d (from 5D) bi-Laplace equation with a δ-source, and its solution (R is 4d distance, radius) is easy to find:

$$\triangle_{(4)}^2 \varphi = -\frac{a}{R^3}\,\delta(R); \;\; \varphi(R^2) = \frac{a}{8}\ln R^2 - \frac{b}{R^2} \;\; (+\,c\,, \text{ but } c \text{ does not matter}); \tag{23}$$

the force between two point masses is $F_{\text{point}} = \frac{a}{4R} + \frac{2b}{R^3}$ (a, b are proportional to both masses).

Now let us suppose that all masses are distributed along the extra dimension with a *universal function*, $\mu(p)$, $\int \mu(p)\,dp = 1$. Then the attracting (gravitational) force takes the next form [see (23); r is usual 3d distance; note that $V(r)$ can be restored if $F(r)$ is measured]:

$$F(r) = \frac{d}{dr}\iint_{-\infty}^{\infty} \varphi(r^2 + (p-q)^2)\,\mu(p)\,\mu(q)\,dp\,dq = \frac{a\,r}{4}\,V - b\,V\,', \;\; V(r) = \iint \frac{\mu(p)\,\mu(q)\,dp\,dq}{r^2 + (p-q)^2}. \tag{24}$$

Taking $\mu_1(p) = \pi^{-1}/(1+p^2)$ (typical scale along the extra dimension is taken as unit, $L = 1$), one can find $rV_1(r) = 1/(2+r)$ and (note, if $a = b$, the Rindler term, $\sim r^0$, vanishes)

$$F(r) = \frac{a}{8+4r} + \frac{2b(1+r)}{r^2(2+r)^2} = \frac{b}{2r^2} + \frac{a(2+r) - 2b}{4(2+r)^2}.$$

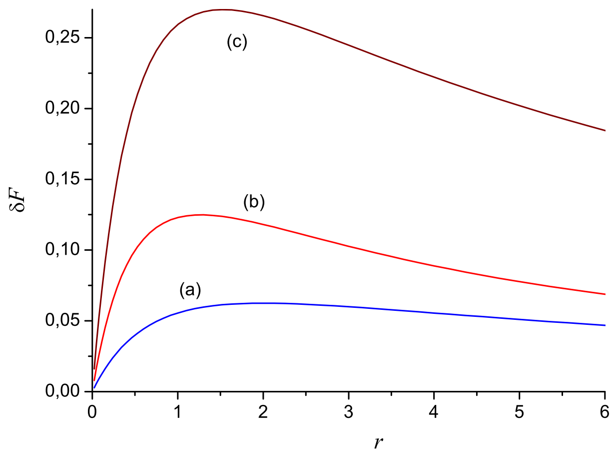

Fig. 1: curve (a) shows $\delta F = F - 1/r^2$, see eq. (24) and text below [$a{=}b\,({=}2)$ is chosen to ensure $\delta F(0){=}0$; more exactly, this is the dimensionless deviation from Newton's law, $(F/F_N - 1)L^2/r^2$ as a function of r/L]; curves (b), (c) correspond to $\mu_2{=}2\pi^{-1}/(1{+}p^2)^2$, $\mu_3{=}2\pi^{-1}p^2/(1{+}p^2)^2$; a/b is chosen also that to ensure $\delta F(0){=}0$.

We see that, in principle, this theory can explain galaxy rotation curves, $v^2(r) \sim rF \overset{r\to\infty}{\longrightarrow}$ const, without need for Dark Matter or MOND.

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[6] The superposition principle emerges due to (a) the huge size L of quanta along the extra-dimension and (b) the fact that f-waves are almost tangential in the shell [so, some scattering amplitudes (framing vectors or something similar) of different parts along the extra dimension, with the same projection (i.e. cophased), should be summed up].

However, at this stage it is not easy to give an estimate for the length scale L: it depends on the mass distribution both along the extra dimension and in the ordinary space. Note, for example, that the usual feature of Newton's law of gravity, that a spherical massive shell has no effect on inner massive bodies, is no more true.

Generation of the most representative components of Riemann tensor (i.e., gravitational waves, GW) is described, a bit schematically, by the next equation [see eq-s (7), (17); α, β – space indexes; full space derivatives, $()'$, do not matter in the RHS]:

$$\Box R_{0\alpha 0\beta} \simeq \ddot{R}_{\alpha\beta} + ()'; \quad \ddot{R}_{\alpha\beta} \sim T_{\alpha\beta} + \ddot{B}_{0\alpha 0\beta}(\Lambda^{\bullet 2}) + ()', \quad \text{while in GR:} \quad \sim \lambda^2_{\text{Planck}} \ddot{T}^{(\text{phen.})}_{\alpha\beta} + ()'. \quad (25)$$

So, one can suggest that, while $L^2 \ddot{T}_{\alpha\beta} \gg T_{\alpha\beta}$ (that is, for short waves, $\lambda \ll L$), generation of gravitational waves, by virtue of $T_{\mu\nu}$, in the new gravity is much 'weaker' than in GR.

However, it seems that the giant polarizations can also contribute to the process (GW generation). They should form a kind of halo, a disturbance of size L, near a heavy body; the form of this halo is either cusped or cored – depending on presence or absence of the Rindler term.

The 00-component of the symmetric part (16) gives an equation for the gravitational potential, as follows (neglecting differentiation along the extra dimension, near the middle of the shell; the static problem):

$$\triangle_{(3)} \varphi \sim < \Lambda^{\bullet 2} >_{00} .$$

Near a body, the non-Newtonian part of potential behaves like

$$\text{either} \quad \delta\varphi \sim r, \quad \text{hence} \quad < \Lambda^{\bullet 2} >_{00} \sim 1/r \ \text{(cusped)},$$

$$\text{or} \quad \delta\varphi \sim r^2, \quad \text{hence} \quad < \Lambda^{\bullet 2} >_{00} \sim \text{const} \ \text{(cored)};$$

in any case, at large scales, $r > L$, the halo drops: $\delta\varphi \sim \ln r$, hence $< \Lambda^{\bullet 2} >_{00} \sim 1/r^2$.

The absence of divergency (no cusp, but core) seems a natural requirement,[7] and in this case the generation of short GW, by virtue of such a halo, is also suppressed (exponentially).

This difference between these two gravities, with respect to generation of GW, can in principle be tested: the method based on pulsar timing, that to observe very long (nHz) gravitational waves, is actively discussing (see e.g. [7]).

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[7]There is a subtlety here; the force includes two integrations along the extra-dimension, while the potential only one (so, some small Rindler term still may survive).