Conditions for stable equilibrium of the upright human body in the presence of external influence

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Introduction: This paper is a continuation of the earlier work, which considered the stable equilibrium of upright human body in the absence of external factors.
External forces are introduced herein, which may disturb the stable equilibrium and cause the body to tumble. The paper also introduces the concept of the coefficient of the stable equilibrium and proposes a formula for its calculation. Finally, a formula is set up to obtain the maximum value of external forces under which the body still maintains the state of equilibrium.

Key words: stability coefficient, force of gravity, external force, overturning.

The previous paper addressed the situation with no external force. For the sake of simplicity, we introduce the coefficient of stability as applied to a stationary ankle joint (special footwear in sports where skis or skates are used provide such relative immobility of ankle joint):

$$K_{st.} = \frac{M_{res}}{M_{tur}} \qquad (1),$$

where M_{res} – resisting moment of gravity

M_{tur} – moment (torque) of external force

If $K_{st.} \geq 1$, body maintains state of equilibrium, while if $K_{st.} < 1$ – it loses equilibrium and overturns.

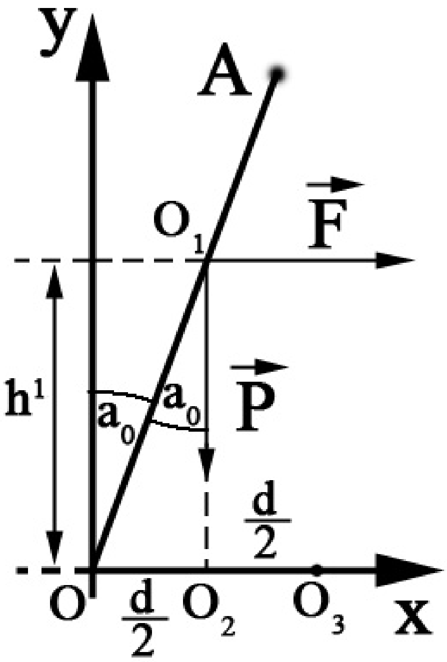

Fig. 1. Schematic representation of human body in a position with the maximum degree of stability [1].

It is clear from Fig. 1, that $M_{res} = |\vec{P}| \cdot \frac{d}{2}$ and $M_{tur} = |\vec{F}| \cdot h'$, when point O3 is the center of rotation. External influences are represented by $|\vec{F}|$, which is the horizontal effect of all external forces. It is directed towards the gradient of a human body as applied to point O1 - common center of mass (CCM) of a human body.

Explanation of fig.1:

$|OA| = H$ – human height;

$|OO_1| = h$ – level of body's common center of mass;

$|OO_3| = d$ – foot length;

$\vec{P}$ – force of gravity on human body, including vertical effect of external forces (body weight);

$\vec{F}$ – horizontal component of resultant external forces;

O_1 – body's common center of mass

O_2 – geometrical center of the area of a foot

O_3 – body hinge point;

α_0 – Optimum slope angle of a human's body bending forward, at which the maximum degree of stability of equilibrium is achieved [1].

With respect to the center of rotation O_3 (Fig. 1) the following is true:

$$M_{res} = |\vec{P}| \cdot \frac{d}{2} \quad \text{and} \quad M_{tur} = |\vec{F}| \cdot h'$$

As $d = 2 \cdot h \cdot \sin\alpha_0$ and $h' = h \cdot \cos\alpha_0$, we receive:

$$M_{res} = |\vec{P}| \cdot h \cdot \sin\alpha_0 \quad \text{and} \quad M_{tur} = |\vec{F}| \cdot h \cdot \cos\alpha_0 \qquad (3)$$

By simple transformations we receive:

$$K_{st} = \frac{|\vec{P}|}{|\vec{F}|} \cdot tg\,\alpha_0 \qquad (4)$$

Paper [1] defined the optimum angle α_0 for men:

$$\alpha_0 = arc\sin\left(0{,}44\frac{d}{H}\right) \qquad (5)$$

And for women:

$$\alpha_0 = arc\sin\left(0{,}48\frac{d}{H}\right) \qquad (6)$$

Considering (5) and (6) the formula (4) will result in the following:

$$K_{st} = \frac{|\vec{P}|}{|\vec{F}|} \cdot \mathrm{tg}\left[arc\sin\left(0{,}44\frac{d}{H}\right)\right] \qquad (7) \quad \text{for men}$$

$$K_{st} = \frac{|\vec{P}|}{|\vec{F}|} \cdot \mathrm{tg}\left[arc\sin\left(0{,}48\frac{d}{H}\right)\right] \qquad (8) \quad \text{for women}$$

When $K_{st} = 1$ $\quad |\vec{F}| = |\vec{F}|_{max}$ (9)

$|\vec{F}|_{max}$ – the maximum value of the external force at which body maintains in a state of equilibrium;

When $|\vec{F}| > |\vec{F}|_{max}$ (т.е. $K_{st} < 1$) body overturns.

Considering condition (9), from (7) and (8) we deduce:

$$|\vec{F}|_{max} = |\vec{P}| \cdot \mathrm{tg}\left[arc\sin\left(0{,}44\frac{d}{H}\right)\right] \qquad (10) \quad \text{for men}$$

$$|\vec{F}|_{max} = |\vec{P}| \cdot \mathrm{tg}\left[arc\sin\left(0{,}48\frac{d}{H}\right)\right] \qquad (11) \quad \text{for women}$$

From these formulae we can obtain numerical values of $|\vec{F}|_{max}$ for human bodies of different parameters.

Based on the above, we can conclude:

1. When $|\vec{F}| \leq |\vec{F}|_{max}$ body maintains its balance;
2. When $|\vec{F}| > |\vec{F}|_{max}$ body overturns;
3. Given the same external effects and identical anthropometry, stability in women is greater than in men.

Literature:

1. K. Moistsrapishvili. Once Again the Equilibrium Stability of a Man. Intellectual Archive. 2016.